A Network-Based Parking Model for Recurrent Short-Term Trips

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Abstract

Efficient parking management strategies are vital in central business districts of mega-cities where space is restricted and congestion is intense. Variable parking pricing is a common parking management strategy in which vehicles are charged based on their dwell time. In this paper, we show that road pricing and variable parking pricing are structurally different in how they influence the traffic equilibrium with elastic demand. Whereas road pricing strictly reduces demand, parking pricing can reduce or induce demand. Under special scenarios, the demand only increases with respect to parking price when parking dwell time time shows increasing returns to scale with respect to the variable parking price. The emergent traffic equilibrium with parking is formulated as a Variational inequality and a heuristic algorithm is presented to find the solution. Numerical experiments are conducted on two networks. Analysis of the first network, with elastic demand and variable parking capacity, shows that parking capacity is only influential in the equilibrium when the variable parking price is low. The second network, a grid network with fixed demand and parking capacity, depicts a larger parking search time at the center of the network with parking zones that are accessible more travelers.

Keywords: Parking; Pricing; Traffic assignment; Variational Inequality

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1 Introduction

Efficient parking management strategies are vital in Central Business Districts (CBDs) of mega-cities where space is restricted and congestion is intense. A great deal of parking demand in these regions is dedicated to travelers who need to visit their final destination for some specified period called dwell time before returning to their origin location (Anderson and de Palma, 2007). Shopping trips in CBDs are an example of these trips. Travelers who engage in such parking behavior incur a cost which is comprised of traveling to a chosen parking area, searching for a spot, paying the parking fee, and walking to the final destination. In day-to-day equilibrium conditions or in the presence of information systems such as mobile apps, travelers adjust their trips to minimize their costs. This adjustment includes choosing an affordable parking area in the vicinity of the final destination. Parking areas are underground or multi-floor parking garages, surface lots, or a collection of on-street parking spots. They can be public or private and generally require a fee admission which can include both a fixed and a variable price component. The variable price component plays a key role in parking management as its impact on parking behavior is twofold. First, increasing the variable fee of a particular parking area increases costs and can directly abate demand. Second, the same increase in the parking fee motivates travelers to shorten their dwell time which can lead to lower parking occupancy, lower searching time and cost, and finally higher demand. The role of variable parking pricing is amplified in the presence of multiple public and private parking management authorities who are generally in competition with each other (Anott and Rowse, 2009; Arnott and Rowse, 2013). This paper investigates the role of time-based parking pricing (hereafter referred to as variable parking pricing) on traffic equilibrium conditions and parking search time.

Parking studies are broadly categorized based on modeling framework, search mechanism, and context. The two main modeling frameworks include simulation and analytic approaches. Simulations capture complex dynamics of parking but require detailed data for calibration. Often, lack of sufficient data is justified through applying behavioral rules which are mostly inconsistent among different studies (Benenson et al., 2008; Gallo et al., 2011; Nourinejad et al., 2014). In Benenson et al. (2008), for instance, parking seekers relinquish their on-street parking search after some time threshold (10 minutes) and head for off-street parking instead and in Nourinejad et al. (2014), parking seekers start the cruising process when within 500 meters of their final destination. In comparison, analytic models, with a few exceptions, are less data-hungry and insightful but are generally aggregate and not amenable to detailed results (Arnott and Inci, 2006; Arnott and Rose, 1999; Anderson and de Palma, 2004). In Arnott and Inci (2006), for instance, a parking model is developed for downtown areas with equal-sized blocks and a constant demand over the region. Although aggregate, the model provides very useful insights such as "it is efficient to raise the on-street parking fee to the point where cruising for parking is eliminated without parking becoming unsaturated". More recently, there is growing advocacy for network-based analytical models that allow for a finer level of policy-making. Two such studies of such, to our knowledge, are Boyles et al. (2014) and Qian and Rajagopal (2014). Both studies develop an equilibrium assignment of parking seekers to spatially disaggregate parking areas but use different search mechanisms.

Searching mechanisms are divided into zone-based[1] searching and link-based searching. In zone-based searching, seekers only start searching for a spot when they reach a zone and each zone is associated with a search time which is assumed to be a function of the zones occupancy (i.e. ratio of the total number of parked vehicles over the total number of available spots) (Qian and Rajagopal, 2014). Applications of zone-based searching are not limited to parking. In taxi equilibrium models, taxi drivers search for passengers in different zones and incur a searching cost which is generally assumed to be a function of the total number of searching taxis and passengers in that zone. Taxi searching time is usually lower with more passengers and less taxis in each zone (Yang and Wong, 1998; Yang et al., 2002; Yang et al., 2010a; Yang et al., 2010b; Yang and Yang, 2011). In link-based searching, seekers search for a spot in any of the links that are on their route to a final destination zone. One of the interesting implications of a link-based search model, as is shown in Boyles et al. (2014), is the smooth transition of the vehicles from "driving" to "searching for parking" which is inherent in the equilibrium structure of the model. The computational load of the model, however, hinders its power in policy-making.

Parking studies can also be classified based on context into zero and non-zero turnover rate models. Turnover refers to the rate at which vehicles leave a parking area which is also an indication of parking duration (also known as parking dwell time). Hence, zero turnover parking indicates that vehicles only enter parking areas without leaving. This type of parking is common in the morning commute context where the one's major concern is the dynamic arrival pattern of the vehicles at the parking zones. These studies are usually defined for stylized settings such as a single bottleneck linear city (Zhang et al., 2008; Qian et al., 2012) or a parallel bottleneck city with several corridors (Zhang et al., 2011). A more general network-based zero turnover model is developed by Qian and Rajagopal's (2014). Non-zero turnover models are more appropriate for short duration activities such as shopping. In these models, one is concerned with both the arrival and departure rate of vehicles from each parking area. Under steady-state conditions, the arrival rate should be equal to the departure rate of vehicles from each parking area (Arnott, 2006; Arnott and Rowse, 2009; Arnott an Inci, 2010; Arnott and Rowse, 2013; Arnott, 2014; Arnott et al., 2015). In non-zero turnover simulation models such as Guo et al. (2013) and Nourinejad et al. (2014), the sum of vehicles entering and leaving each parking area are equal.

The policy implications of parking have also been the subject of many studies. Among the more innovative ones are parking permit schemes which involve distributing a number of permits between travelers and restricting vehicles to spend the permits for parking (Zhang et al., 2011; Liu et al., 2014). In another novel policy, He et al. (2015) study the optimal assignment of vehicles to parking spots while considering the competition game between the vehicles. The authors show the existence of multiple equilibria and propose a robust pricing scheme. Qian and Rajagopal (2014) study parking pricing strategies using real-time sensors to manage parking demand. Using parking pricing and information provision systems, Qian and Rajagopal (2014) propose a dynamic stabilized controller to minimize the total travel time in the system. Parking prices are then adjusted in real-time according to occupancy information collected from parking sensors.

[1] By zone, we refer to either an off-street parking lot or a collection of on-street parking spaces.

In this paper, we present a non-zero turnover, zone-based search, analytical model for parking systems. Given the non-zero turnover rate, we consider both arrival and departure rates of vehicles to parking areas which are assumed to be equal under steady-state conditions. Our model is therefore distinguished from Qian and Rajagopal (2014) which is a zero turnover model. Contrary to Boyle et al. (2014), we use the zone-based search mechanism which, due to its simplicity, helps derivation of the analytical results and improves policy evaluation. The presented model is also distinguished from the analytical models of Arnott and Inci (2006), Arnott (2014), and Arnott et al. (2015) since it considers the topological network of parking.

We particularly focus on unassigned parking where drivers have to cruise to find a spot. These trips have shorter dwell times and belong to frequent drivers such as shoppers. We study cases where parking supply can be varied and cases where parking supply is fixed and exogenous and represented by zones in a network. Each parking zone has a specified capacity and can either be an off-street lot or a group of on-street parking spots. The modeled network is assumed to be a CBD where travelers reside far away. This assumption is previously imposed by Anderson and de Palma (2004) as well.

The remainder of this paper is organized as follows. The model is presented in Section 3. Equilibrium conditions are discussed in Section 4. Parking competition is explained in Section 5. Examples are provided in Section 6. Conclusions are presented in Section 7.

2 The model

2.1 The network

Consider a transportation network $G(N, A)$ with node and arc sets N and A, respectively. To model the parking process we further partition the node set N into external nodes denoted by R, parking zones denoted by I, and internal zones denoted by S so that $N = R \cup I \cup S$. Let $R = \{1,..,r,..,|R|\}$, $S = \{1,..,s,..,|S|\}$, and $I = \{1,..,i,..,|I|\}$. External zones are the origin location of travelers (say home) and internal zones are their destination zones (say a shopping center). Each traveler is associated with one external zone and one internal zone and can park at any of the parking zones. For modeling non-zero turnover parking, we consider two types of trips called inbound and outbound. Inbound trips involve travelers who leave an external zone from where they drive to a parking zone. After parking, the inbound traveler walks from the parking zone to an internal zone as is shown in Fig. 1a. Hence, the path of every inbound traveler includes the sequence $r \to i \to s$. Each inbound trip is associated with a return outbound trip. That is, outbound trips are the reverse direction of inbound trips. The path of every outbound traveler includes the sequence $s \to i \to r$. Fig. 1a depicts the general inbound and outbound trip trajectories and Fig. 1b illustrates an example of internal and external zones where the internal zones represent destination locations in the CBD of a city and the external zones represent the gateways to the CBD. Let us also partition the link Set A into A_d and A_w representing the driving and walking links, respectively, as is also shown in Fig. 1a.

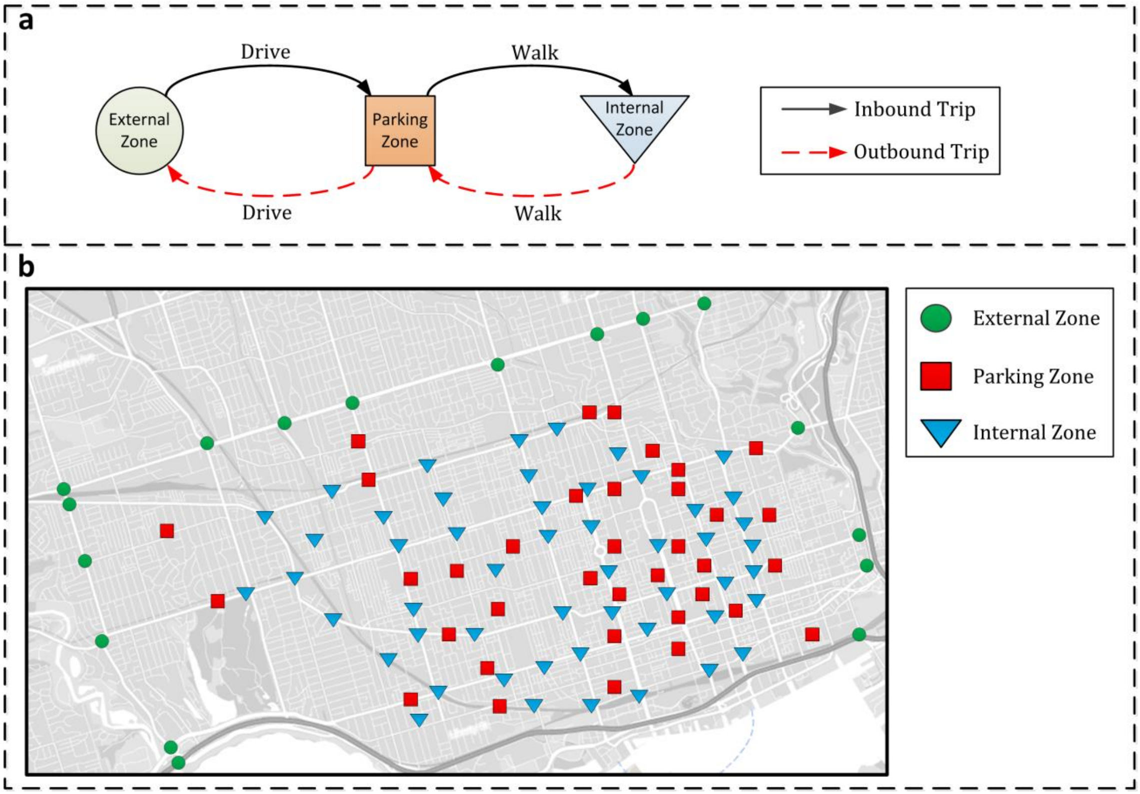

Fig. 1. a: Inbound and outbound trip trajectories; b: An example for the three zone types.

The following sets are now defined. Let $V = R \times S$ represent the set of Origin-Destination (O-D) pairs. For each O-D pair (r, s), let $\Omega(r, s)$ be the set of parking zones that are within the parking zone choice-set of these travelers. The Set $\Omega(r, s)$ can be defined according to features such as walking distance from parking *i* to destination *s* and the cost of parking. Clearly, parking zones that are too far from the internal destinations zones are less likely to be included in the choice set. For every O-D pair $(r, s) \in V$ and parking zone $i \epsilon \Omega(r, s)$, let $\psi(r, s, i)$ represent the set of routes for the segments of the journey which include driving links. Each route is comprised of a set of driving links connecting the zone *r* to *i* and zone *i* to *s*. For instance, in Fig. 1a there is only one route of this nature which includes the following sequences of zones: $r \rightarrow i \rightarrow s$.

Let $d_{rs,a}^{i}$ denote the flow of vehicles belonging to O-D pair $(r, s) \in V$ who choose parking $i \epsilon \Omega(r, s)$ via route $a \epsilon \psi(r, s, i)$. Let x_b be the flow and $\tau_b(x_b)$ the travel time of driving link $b \in A_d$ and let w_b represent the walking time on link $b \in A_w$. It is commonly assumed that the travel time on driving link $b \in A_d$ is a continuous and monotonically increasing function of link flow x_b and the travel time on walking link $b \in A_w$ is independent of the flow. Let $\triangle$ represent the path-link incidence matrix where $\triangle_{a,b} = 1$ if link $b \in A_d$ is included in route $a \in \psi(r, s, i)$ and $\triangle_{a,b} = 0$, otherwise. Hence we have $x_b = \sum_{(r,s) \in V} \sum_{i \epsilon \Omega(r,s)} \sum_{a \in \psi(r,s,i)} d_{rs,a}^{i} \triangle_{a,b}$.

2.2 The non-zero turnover parking process

The parking search process is explained in this section. First, the following assumption is imposed:

Assumption 1: Under equilibrium conditions travelers will choose to park at a zone with the lowest perceived cost.

Assumption 1 is justified under at least two conditions. First, if the trips are recurrently performed, travelers become familiar with the process and choose to park at a zone with the lowest generalized cost. Second, when parking information such as parking occupancy is provided to users via apparatus such as mobile apps, travelers are better informed about which zone to choose for parking. In essence, Assumption 1 implies that travelers will not hop between parking zones and will instead choose the one with the lowest perceived cost. The cost of parking is comprised of the cost of traveling from the external zone to a parking zone, the cost of searching for parking, the parking fee which can include both a fixed and a variable component, the cost of walking from the parking area to the internal zone, the cost of walking from the internal zone to the parking area, and the cost of driving from the parking area to the origin zone.

Using Assumption 1, we can now analyze the parking pattern of travelers. Let d_{rs}^{i}, $\forall (r, s) \in V$, $i \in \Omega(r, s)$, represent the flow of vehicles that originate at zone *r*, terminate at zone *s*, and park at zone *i* and let $d_{rs} = \sum_{i \in \Omega(r,s)} d_{rs}^{i}$ represent the total flow from *r* to *s*. We assume that all travelers belonging to the origin-destination pair (r, s) who choose parking *i* will remain there for a period of h_{rs}^{i} called the dwell time. This assumption is justified as travelers belonging to the same origin-destination pair are likely to be homogenous (Yang and Huang, 2005).

Let q_i be the total occupancy of parking $i \in I$ under equilibrium and let k_i be the capacity of parking *i* measured in vehicles. Note that k_i is a given whereas q_i is obtained from the equilibrium. Given the flow of vehicles and their dwell time, we have:

$$q_i = \sum_{(r,s)} d_{rs}^{i} h_{rs}^{i} \qquad \forall i \in I \qquad (1)$$

Parking search time is typically assumed to be a convex function of parking occupancy q_i and capacity k_i (Axhausen et al., 1994; Anderson and de Palma, 2004; Levy et al., 2012; Qian and Rajagopal, 2014). The general form of this function $F_i(q_i)$ as explained in Axhausen et al. (1994) is:

$$F_i(q_i) = \frac{l_i \mu_i}{1 - \frac{q_i}{k_i}} \qquad \forall i \in I \qquad (2)$$

where l_i is the is the average searching time in parking area *i* when occupancy is low or medium and μ_i is a constant representing how drivers adopt occupancy information. When $\mu_i = 0$, drivers are unaware of the searching time and when $\mu_i = 1$ drivers are completely aware of searching time. Axhausen et al. (1994) estimated the search function with a coefficient of determination $R^2 = 0.91$ for Frankfurt, Germany. The searching time function $F_i(q_i)$ asymptotically goes to infinity as q_i approaches k_i, i.e. $lim_{q_i \to k_i} F_i(q_i) = \infty$. This implies that a driver entering a full occupancy parking will never find a spot.

2.3 Generalized travel costs

The cost of parking $i \in I$ is assumed to consist of a fixed cost g_i measured in dollars and a variable cost of p_i dollars per each hour of dwell time. Hence, for the O-D pair (r, s) a traveler who chooses parking *i* will incur a total of $g_i + p_i h_{rs}^i$ dollars in parking costs. Let $P = \{(p_1, g_1) \dots, (p_i, g_i), \dots, (p_{|I|}, g_{|I|})\}$ be the set of fixed and variable parking costs. We can now derive the generalized travel costs. Let $C_{rs,a}^i$ be the generalized travel cost for travelers of O-D pair (r, s) who choose parking $i \epsilon \Omega(r, s)$ via route $a \epsilon \psi(r, s, i)$. This cost is composed of the following six terms: (i) traveling from external zone *r* to parking *i* via route *a* with a travel time $t_{ri,a}$, (ii) searching for parking for a period of $F_i(q_i)$, (iii) a parking cost of $g_i + p_i h_{rs}^i$ dollars, (iv) walking from parking *i* to zone *s*, (v) walking from zone *s* to parking *i*, and (vi) traveling from parking *i* to external zone *r* via route a:

$$C_{rs,a}^i = \alpha t_{ri,a} + \beta F_i(q_i) + \left(g_i + p_i h_{rs}^i\right) + \gamma w_{is} + \gamma w_{si} + \alpha t_{ir,a}$$
$$\forall (r, s) \in W, \forall i \in \Omega(r, s), \forall a \in \psi(r, s, i) \qquad (3)$$

In Eq. 3, α, β, and γ represent the marginal cost of each hour of travel time, each hour of parking search time, and each hour of walking time, respectively. For the first term on the right-side of Eq. 3, we have $t_{ri,a} = \sum_b \tau_b \, \Delta_{a,b}$.

Using Eq. 3, the corresponding minimum cost via the shortest route for a O-D travelers $(r, s) \in V$ parking at zone $i \in \Omega(r, s)$ is $C_{rs}^i = \min_{a \in \psi(r,s,i)} C_{rs,a}^i$. However, C_{rs}^i only represents the observed cost of O-D pair $(r, s) \in V$ travelers who choose parking $i \in \Omega(r, s)$. Let us also assume an additional unobserved cost of ε_{rs}^i which is independently and identically Gumbel distributed for all parking zones $i \in I$ that can be chosen by travelers of O-D pair (*r*,*s*). With this assumption, the probability that an O-D pair $(r, s) \in V$ traveler chooses parking $i \in \Omega(r, s)$ is denoted by π_{rs}^i which can be obtained using the following logit-based probability function:

$$\pi_{rs}^i = \frac{\exp\left(-\theta C_{rs}^i\right)}{\sum_{j \in \Omega(r,s)} \exp\left(-\theta C_{rs}^j\right)} \qquad \forall i \in \Omega(r, s) \qquad (4)$$

where θ is a dispersion parameter representing the variation in the cost perception of travelers. Note that Eq. 4 benefits from the following assumption:

Assumption 2: Travelers are stochastic in choosing a parking area but deterministic in choosing routes. This assumption is justified due to the availability and accuracy of route-guidance advanced traveler information systems.

We also assume that the O-D pair demand is a continuous and decreasing function of the expected, perceived travel cost of each O-D pair. The O-D pair demand function is denoted by D_{rs} and the expected, perceived travel cost is denoted by η_{rs} for each $(r, s) \in V$. Hence, we have:

$$d_{rs} = D_{rs}(\eta_{rs}) \qquad \forall (r, s) \in W \qquad (5)$$

Given the logit-based parking choice model in Eq. 4, the expected minimum cost for each $(r, s) \in V$ is:

$$\eta_{rs} = E\left(\min_{i \in \Omega(r,s)} \left\{C_{rs}^i\right\}\right) = -\frac{1}{\theta} \ln\left(\sum_{i \in \Omega(r,s)} \exp\left(-\theta C_{rs}^i\right)\right) \qquad \forall (r, s) \in W \qquad (6)$$

2.4 Parking dwell time

Recall that parking dwell time h_{rs}^{i} is the time spent by travelers of O-D pair (r, s) at parking zone $i\epsilon\Omega(r, s)$. The following assumption is now imposed:

Assumption 3: The dwell time of travelers of O-D pair (r, s) at parking zone $i\epsilon\Omega(r, s)$is assumed to be a function of the variable parking cost p_i of parking *i*.

Let $H_{rs}(p_i)$ denote this function which is assumed to be convex and monotonically decreasing with p_i. Moreover, it is also sound to assume that dwell time approaches zero as p_i tends to infinity, i.e. $\lim_{p_i \to \infty} H_{rs}(p_i) = 0$. Hence, we have:

$$h_{rs}^{i} = H_{rs}(p_i) \qquad \forall(r, s) \in W, \forall i \in \Omega(r, s) \qquad (7)$$

According to Eq. 7 and Eq. 3, the variable parking price p_i influences the parking behaviors in two ways. First, increasing the p_i leads to a higher generalized cost of parking at parking area *i* as imposed by the term $p_i h_{rs}^{i}$ in Eq. 3. Second, increasing p_i leads to a lower dwell time as implied by Eq. 7, which can in turn reduce the generalized cost of parking at parking area *i* as imposed by the term $p_i h_{rs}^{i}$ in Eq. 3.

3 Comparative analysis of road pricing and parking pricing

We show here that road pricing and parking fares are structurally different in how they influence the traffic equilibrium. Whereas road pricing reduces demand, parking fares can reduce or induce demand. Mathematically, we have $\frac{\mathrm{dD}}{\mathrm{d}\hat{p}} < 0$ where $\hat{p}$ is the road toll and $\frac{\mathrm{dD}}{\mathrm{d}p} <> 0$ where p is the variable parking price and *D* is the demand function. Consider the network of Fig. 1a which has one origin *r*, one destination *s*, and one parking area *i*. A toll $\hat{p}$ is imposed on the driving link (r, i) and a variable parking price *p* is imposed on parking area *i*. The demand function is defined such that the generated demand is strictly decreasing with respect to the generalized cost, i.e. $\frac{\mathrm{d}\, D_{rs}(\eta_{rs})}{\mathrm{d}\, \eta_{rs}} <$ 0.

The following two lemmas are now defined and later used to prove Proposition 1.

Lemma 1: Demand is strictly decreasing with respect to the road toll, i.e. $\frac{\mathrm{dD}}{\mathrm{d}\hat{p}} < 0$.

Proof:

Let us rewrite $\frac{\mathrm{dD}}{\mathrm{d}\hat{p}}$ as

$$\frac{\mathrm{d}D_{rs}}{\mathrm{d}\hat{p}} = \frac{\mathrm{d}\, D_{rs}}{\mathrm{d}\, \eta_{rs}} \cdot \frac{\mathrm{d}\, \eta_{rs}}{\mathrm{d}\, \hat{p}} \qquad (8)$$

It is already assumed that $\frac{\mathrm{d}\, D_{rs}}{\mathrm{d}\, \eta_{rs}} < 0$ as demand is strictly decreasing with respect to the generalized cost. It is also evident that $\frac{\mathrm{d}\, \eta_{rs}}{\mathrm{d}\, \hat{p}} > 0$ because $\hat{p}$ is the out-of-pocket money paid by travelers to traverse road (r, i). Hence, the product of the two terms on the RHS of Eq. (8) is negative and $\frac{\mathrm{d}D_{rs}}{\mathrm{d}\hat{p}} < 0$. ■

Lemma 2: Changing the parking fare may induce or reduce demand, i.e. $\frac{\mathrm{d}D}{\mathrm{d}p} <> 0$.

Proof:

Let $D = D_{rs}$, $h = h_{rs}^{i}$, $\eta = \eta_{rs}$, $k = k_i$, and $\mu = \mu_i$.

Let us rewrite $\frac{\mathrm{dD}}{\mathrm{d}p}$ as

$$\frac{\mathrm{d}D}{\mathrm{d}p} = \frac{\mathrm{d}\,D}{\mathrm{d}\,\eta} . \frac{\mathrm{d}\,\eta}{\mathrm{d}\,p} \tag{9}$$

It is already assumed that $\frac{\mathrm{d}\,D}{\mathrm{d}\,\eta} < 0$ as demand is strictly decreasing with respect to the generalized cost. Hence, we focus on the second term on the RHS of Eq. (9). By taking the derivative of Eq. (3), we have

$$\frac{\mathrm{d}\,\eta}{\mathrm{d}\,p} = \frac{\mathrm{d}\,(hp)}{\mathrm{d}\,p} + \frac{\mathrm{d}\,F}{\mathrm{d}\,p} \tag{10}$$

By taking the derivative of Eq. (2), the second term on the RHS of Eq. (10) can be rewritten as

$$\frac{\mathrm{d}\,F}{\mathrm{d}\,\mathrm{p}} = \frac{\mu[(\mathrm{d}\,h/\mathrm{d}\,p)D + (\mathrm{d}\,D/\mathrm{d}\,\mathrm{p})h]}{k(1 - hd/k)^2} \tag{11}$$

By inputting Eq. (11) into Eq. (10), inputting Eq. (10) into Eq. (9), and simplifying the terms, we have

$$\frac{\mathrm{d}D}{\mathrm{d}p} = \frac{\mathrm{d}D}{\mathrm{d}\eta}\left[\frac{(\mathrm{d}(hp)/\mathrm{d}p) + \omega(\mathrm{d}h/\mathrm{d}p)D}{1 - \omega h(\mathrm{d}D/\mathrm{d}\eta\,)}\right] \tag{12}$$

where $\omega = \mu/[k(1 - hD/k)^2] > 0$. Analysis of Eq. (12) concludes the following:

$$\frac{\mathrm{d}D}{\mathrm{d}p} > 0 \qquad if \quad D > D^* \tag{13a}$$

$$\frac{\mathrm{d}D}{\mathrm{d}p} < 0 \qquad if \quad D < D^* \tag{13b}$$

in which $D^* = \frac{-(\mathrm{d}\,hp/\mathrm{d}p)}{\omega\,(\mathrm{d}h/\mathrm{d}p)}$. Eq. (13) shows that marginal change of demand with respect to the variable parking cost depends on the value of the materialized demand D. ■

Lemma 1 has the following two remarks:

Remark 1: The variable parking prices has the same effect as the road toll when travelers dwell time is insensitive to variable parking price.

Proof:

When traveler dwell time time is insensitive to the variable parking cost (i.e. $\mathrm{d}h/\mathrm{d}\,p \to 0$), we have $D^* = \frac{-h}{\omega\,(\mathrm{d}h/\mathrm{d}p)} \to \infty$ which, according to Eq. (13b) indicates, that demand is strictly decreasing with respect to the variable parking price. In other words, when $\mathrm{d}h/\mathrm{d}\,p \to 0$, the variable parking cost has a similar impact on demand as a road toll. ■

Remark 2: Demand is strictly increasing with respect to parking dwell time when parking dwell time is highly elastic to variable parking price.

Proof:

Let $e_p^h \leq 0$ be the parking dwell time elasticity with respect to the variable parking price. Given that $e_p^h = \frac{\partial h}{\partial p}\frac{p}{h}$ and $\frac{\partial (ph)}{\partial p} = h(1 + e_p^h)$, we can rewrite D^* in Eq. (13) as $D^* = -p(1 + e_p^h)/\omega e_p^h$.

Consider the two scenarios where $-1 < e_p^h \leq 0$ and $e_p^h \leq -1$. Under Scenario I when $-1 < e_p^h \leq 0$, we have $D^* > 0$ indicating that the demand both increases and decreases. Under Scenario II when $e_p^h \leq -1$, however, we have $D^* \leq 0$ which according to Eq. (13) indicates that the demand is strictly increasing with respecting to the variable parking price. ■

The following proposition is readily derived from Lemma 1 and 2.

Proposition 1: Whereas road pricing reduces demand, variable parking pricing can reduce or induce demand depending on the values of the materialized demand.

4 Equilibrium conditions

4.1 A variational inequality formulation

In this section, we formulate the equilibrium problem using Variational Inequality (VI). The VI formulation is defined for a given Set $P = \{(p_1, g_1) \dots, (p_i, g_i), \dots, (p_{|I|}, g_{|I|})\}$. Hence, according to Assumption 3, the dwell times of each O-D pair traveler at each parking zone is known as well. Let us define the feasible region Γ of the demands and route flows of the equilibrium model as the following set of equations in which d^i is the flow of vehicles into parking zone $i \in I$ and the variables in brackets are the dual variables.

$\sum_{a \epsilon \psi(r,s,i)} d_{rs,a}^i = d_{rs}^i$	$[u_{rs}^i]$	$\forall (r,s) \in V, \forall i \in \Omega(r,s)$	(14a)
$\sum_{i \in \Omega(r,s)} d_{rs}^i = d_{rs}$	$[\lambda_{rs}]$	$\forall (r,s) \in V$	(14b)
$d^i = \sum_{(r,s) \in V} d_{rs}^i$	$[\delta_i]$	$\forall i \in I$	(14c)
$d_{rs,a}^i \geq 0$	$[\varphi_{rs,a}^i]$	$\forall (r,s) \in V, \forall i \in \Omega(r,s)$	(14d)

Constraints (14a) and (14b) represent conservation of flow, constraints (14c) represent occupancy of parking *i,* and constraints (14d) represent non-negativity of path flows. For clarity, let us now partition the cost $C_{rs,a}^i$ (as shown in Eq. 3) into the following terms:

$$C_{rs,a}^i = \varsigma_{rs,a}^i + \beta F_i(q_i) \qquad \forall (r,s) \in V, \forall i \in \Omega(r,s), \forall a \in \psi(r,s,i) \qquad (15)$$

where $\varsigma_{rs,a}^i = \alpha t_{ri,a} + \gamma w_{is} + \gamma w_{si} + \alpha t_{ir,a} + \left(g_i + p_i h_{rs}^i\right)$ represents the total observed travel cost including the cost of driving from *r* to *i,* walking from *i* to *s*, walking from *s* to *i*, and driving from *i* to *r*. With Eq. (15) defined, the VI program is given as follows. Let $\boldsymbol{d} = \{d_{rs,a}^i, (r,s) \epsilon W, i \in \Omega(r,s), a \in \psi(r,s,i)\}$. Find $\left(\boldsymbol{d}^*, {q_i}^*, d_{rs}^*, d_{rs}^{i^*}\right) \in \Gamma, (r,s) \epsilon W, i \in \Omega(r,s), a \in \psi(r,s,i)$ as the equilibrium solution such that:

$$\sum_{(r,s)\epsilon V}\left(\sum_{i\epsilon\Omega(r,s)}\left(\sum_{a\epsilon\psi(r,s,i)}\varsigma^{i}_{rs,a}(\boldsymbol{d})\left(d^{i}_{rs,a}-d^{i^*}_{rs,a}\right)+\frac{1}{\theta}\ln d^{i^*}_{rs}\left(d^{i}_{rs}-d^{i^*}_{rs}\right)\right)-\frac{1}{\theta}\ln d^{*}_{rs}\left(d_{rs}-d^{*}_{rs}\right)-D^{-1}_{rs}(d^{*}_{rs})(d_{rs}-d^{*}_{rs})\right)+\beta\sum_i F_i({q_i}^*)(d^i-d^{i^*})\geq 0 \qquad \forall((\boldsymbol{d}^*,{q_i}^*,d^*_{rs},d^{i^*}_{rs}))\in\Gamma \qquad (16)$$

The Karush-Kuhn-Tucker (KKT) conditions of the VI program in Eq. 16 are derived as

$$d^{i}_{rs,a}: \quad \varsigma^{i}_{rs,a}(\boldsymbol{d}^*)-u^{i}_{rs}-\varphi^{i}_{rs,a}=0 \qquad \forall(r,s)\in V,\forall i\in\Omega(r,s) \qquad (17)$$

$$d^{i}_{rs} \;:\; u^{i}_{rs}+\delta_i-\lambda_{rs}+\frac{1}{\theta}\ln d^{i}_{rs}=0 \qquad \forall(r,s)\in V,\forall i\in\Omega(r,s) \qquad (18)$$

$$d_{rs} \;:\; \lambda_{rs}-D^{-1}_{rs}(d_{rs})-\frac{1}{\theta}\ln d_{rs}=0 \qquad \forall(r,s)\in V \qquad (19)$$

$$d^{i} \;:\; \beta F_i(q_i)-\delta_i=0 \qquad \forall i\in I \qquad (20)$$

The complementarity conditions include constraints (14a) to (14d) and the following two conditions:

$$d^{i}_{rs,a}\varphi^{i}_{rs,a}=0 \qquad \forall(r,s)\in V,\forall i\in\Omega(r,s),\forall a\in\psi(r,s,i) \qquad (21)$$

$$\varphi^{i}_{rs,a}\geq 0 \qquad \forall(r,s)\in V,\forall i\in\Omega(r,s),\forall a\in\psi(r,s,i) \qquad (22)$$

At equilibrium δ_i is interpreted as the cost of searching at parking area *i* as per Eq. (29) and u^{i}_{rs} is interpreted as the minimum generalized travel (both driving and walking) cost of O-D pair $(r,s)\epsilon W$ travelers parking at zone *i* as per (Eq. 17). We now show that the presented VI in equivalent to the equilibrium conditions of Section 2.

First, assume that demand is always non-negative $d^{i}_{rs,a}>0$, so that $\varphi^{i}_{rs,a}=0$ as per Eq. (21). Given that $\varphi^{i}_{rs,a}=0$, by applying the exponential function to both side of Eq. (18) and simplifying the terms, we have

$$d^{i}_{rs}=\exp(-\theta(u^{i}_{rs}+\delta_i-\lambda_{rs}) \qquad \forall(r,s)\in V,\forall i\in\Omega(r,s) \qquad (23)$$

Using Eq. (14b), (Eq. 23) can be rewritten as:

$$\sum_i d^{i}_{rs}=\exp(\theta\lambda_{rs})\sum_i\exp(-\theta(u^{i}_{rs}+\delta_i)=d_{rs} \qquad \forall(r,s)\in V \qquad (24)$$

Thus,

$$\exp(\theta\lambda_{rs})=\frac{d_{rs}}{\sum_i\exp(-\theta\left(u^{j}_{rs}+\delta_j\right))} \qquad \forall(r,s)\in V \qquad (25)$$

Substituting Eq. (25) into Eq. (23) gives

$$d^{i}_{rs}=\frac{\exp(-\theta(u^{i}_{rs}+\delta_i)}{\sum_i\exp(-\theta(u^{j}_{rs}+\delta_j))}d_{rs} \qquad \forall(r,s)\in V,\forall i\in\Omega(r,s) \qquad (26)$$

in which the term δ_i can be related to the cost of searching at parking area *i*. This relevance makes Eq. (26) equivalent to the logit-based choice probability indicating that $d^{i}_{rs}=\pi^{i}_{rs}d_{rs}$.

Eq. (19) can also be reorganized as

$$\lambda_{rs} = \frac{1}{\theta}\ln d_{rs} - \frac{1}{\theta}\ln \sum_i \exp(-\theta(u_{rs}^i + \delta_i) \qquad \forall(r,s) \in V \tag{27}$$

Substituting Eq. (27) into Eq. (25) gives:

$$D_{rs}^{-1}(d_{rs}) = -\frac{1}{\theta}\ln \sum_i \exp(-\theta(u_{rs}^i + \delta_i) \qquad \forall(r,s) \in V \tag{28}$$

which is equivalent to Eq. (6) representing the demand function.

We have shown that the solution of the VI program satisfies all the functional relationships that are required by the parking model as defined in Section 2. The VI program has at least one solution when its feasible region is a compact convex set and the function of the VI as shown is continuous in the feasible region Γ. Given that feasible region Γ as defined in Eq. (14) is a set of linear constraints with non-negativity and given that the VI function in Eq. (16) is continuous within the feasible region, we conclude that the VI has at least one solution (Florian, 2002).

4.2 Solving for equilibrium

An extensive review of solution algorithms for finding the traffic equilibrium is presented by Patriksson (2004). To solve the VI, we use a method in which the traffic flows to parking areas $(d^i, \forall i)$ are first calculated to find the parking search time. Calculating the parking search time can lead to infeasible solutions when the computed parking occupancy is larger than the parking capacity, i.e. $q_i \geq k_i$. To rectify this issue, the parking search time in Eq. (2) is replaced with the following BPR-type equation.

$$F_i(q_i) = l_i\mu_i\left[1 + \left(\frac{q_i}{k_i}\right)^{\vartheta}\right] \tag{29}$$

in which l_i is the is the average searching time in parking area *i*, μ_i is a constant representing how drivers adopt occupancy information, and ϑ is a calibration parameter. The computed parking search time is then used to find the generalized cost and the origin-destination demand. The algorithm terminates upon convergence. The steps of the algorithm are the following:

Step 1. Initialization

Set the iteration number $\upsilon = 0$. Select an initial feasible solution $\boldsymbol{d}^{\upsilon}$**.** The feasible solution can be obtained by setting all travel times equal to free-flow travel times and setting the parking search time equal to zero for all parking areas.

Step 2. Computation of generalized costs

First, using $\boldsymbol{d}^{\upsilon}$**,** find the flow of vehicles into each parking area. The product of vehicle flow into each parking area and the parking dwell times (obtained for a given parking price) gives the parking occupancy which can be used as input in Eq. (29) to find the parking search time of each parking area. Second, using $\boldsymbol{d}^{\upsilon}$**,** find the travel times and the generalized costs as per Eq. (3).

Step 3. Direction finding

Perform a stochastic network loading procedure on the current set of link travel times. This yields and auxiliary link flow pattern $\widehat{\boldsymbol{d}}$**.**

Step 4. Method of successive averages

Using the demand obtained from Step 3, find the new flow pattern by setting

$$\boldsymbol{d}^{v+1} = \frac{v-1}{v}\boldsymbol{d}^{v} + \frac{1}{v}\widehat{\boldsymbol{d}} \tag{30}$$

Step 5. Convergence test

Terminate if the following condition is satisfied with $\varkappa$ being a small number. Otherwise, set $v \rightarrow v+1$ and go to Step 2.

$$\frac{\sqrt{\sum(\boldsymbol{d}^{v+1}-\boldsymbol{d}^{v})^2}}{\sum \boldsymbol{d}^{v}} \leq \varkappa \tag{31}$$

5 Market regimes

Let us first assume that a single operator is in charge of managing all the parking facilities. This operator can be either a public or a private entity. In such cases, the two objective functions of interest are profit maximization (denoted by *PM*) and social surplus maximization (denoted by *SS*). The former can be associated to the private and the latter to public authorities. The profits of collecting parking fees can be define as:

$$PM = \sum_{(r,s)\in V}\sum_{i\in\Omega(r,s)}\left[\left(p_i h_{rs}^{i} + g_i\right)d_{rs}^{i}\right] - \sum_{i\in \mathrm{I}} k_i\sigma_i \tag{32}$$

where the first term on the right represents the generated revenue from parking and the second term represents the total maintenance cost of all parking spots with σ_i denoting the maintenance cost of one parking spot at parking zone $i \in I$. The maintenance cost is not necessarily the cost of physical rehabilitation and can include other supervisory costs such as the cost of parking enforcement for on-street parking. The second objective function is social surplus which can be calculated as:

$$SS = \sum_{(r,s)\in V}\int_0^{d_{rs}} D_{rs}^{-1}(z)dz - \sum_{i\in \mathrm{I}} k_i\sigma_i \tag{33}$$

where $D_{rs}^{-1}(z)$ represents the inverse of the demand function. With the two objective functions, we can now define the following three markets: (i) monopoly, (ii) first best, and (iii) second best. Let us assume for now that the parking operator has monopoly rights and can simultaneously decide on the capacity and the fee structure of all parking zones. Under this market, the objective is to maximize the total profit as shown in Eq. 32. Alternatively, in the first-best market, the objective is to maximize social surplus. Finally, under the second-best market, the objective is to maximize social welfare while ensuring profits are nil. Hence, under the second-best market we have:

$$maximize \;\; \sum_{(r,s)\in V}\int_0^{d_{rs}} D_{rs}^{-1}(z)dz - \sum_{i\in \mathrm{I}} k_i\sigma_i$$

subject to

$$\sum_{(r,s)\in V}\sum_{i\in\Omega(r,s)}\left[\left(p_i h_{rs}^i + g_i\right)d_{rs}^i\right] = \sum_{i\in \mathrm{I}} k_i\sigma_i \tag{34}$$

6 Numerical experiments: the case of the City of Toronto

Numerical experiments are performed first on a network with elastic demand and variable parking capacity and second on a network with fixed demand and fixed parking capacity.

6.1 First network: elastic demand and variable parking capacity

We analyze a simple example to visually present the three defined market regimes of Section 5. Consider the network in Fig. 1a with one O-D pair (r,s) and one parking zone $i \in \Omega(r,s)$. Let $\alpha = \beta = 10$ dollars per hour, $g = 0.5$ dollars, $w_{si} = w_{is} = 0$ hours, $\gamma = 0$ dollars per hour, $\theta = 1$, $\mu_i = 1$, and $l_i = 3$ minutes. The functions are defined as follows. Let $t_{ri} = t_{ir} = 0.5 + \frac{x^2}{1000}$ measured in hours where x is the total demand which is obtained from the demand function $x = D_{rs}(\eta_{rs}) = 20 - \eta_{rs}$. The dwell time function is $H_{rs}(p_i) = 3p_i^{-0.4}$.

The iso-profit and iso-social surplus contours are depicted in Fig. 2 for the simple example. As illustrated the monopoly equilibrium occurs at the optimum of the profit objective function and the first-best equilibrium occurs at the optimum of the social surplus contours. The second-best equilibrium has to lie on the zero profit line where social surplus is maximized.

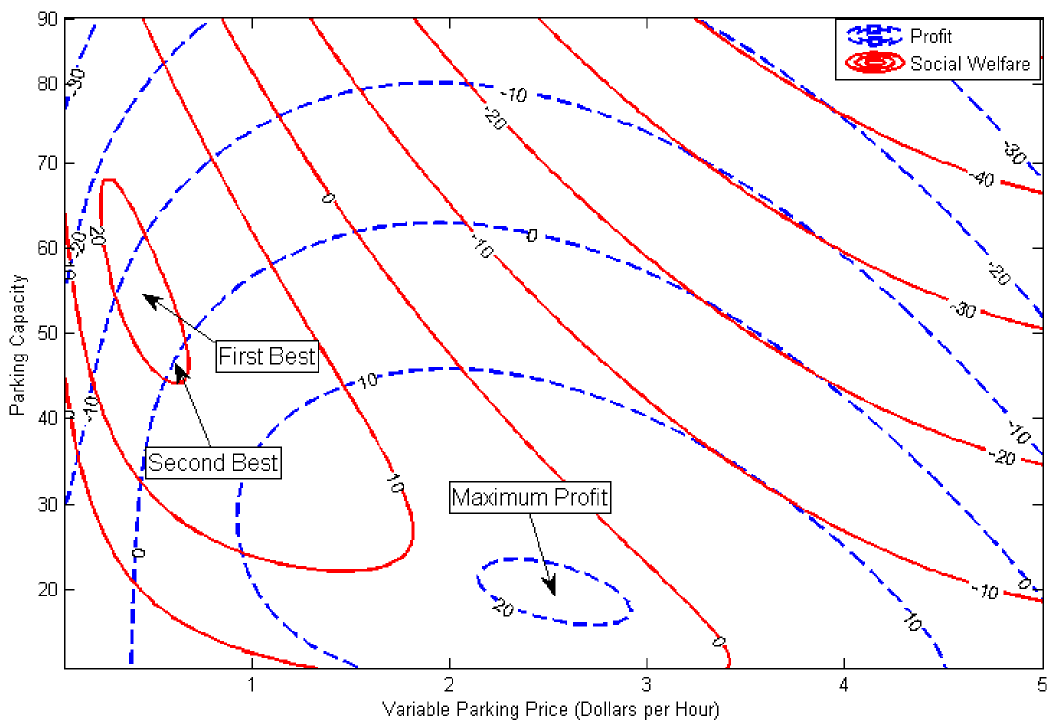

Fig. 2. Iso-profit and iso-social surplus contours for the simple example.

We further investigate the generated profits by considering the following three dwell time function scenarios:

I- $H_{rs}(p_i) = 3p_i^{-1}$: Constant returns to scale, $e_p^h = 1$

II- $H_{rs}(p_i) = 3p_i^{-0.4}$: Decreasing returns to scale, $e_p^h = 0.4$

III- $H_{rs}(p_i) = 3p_i^{-1.4}$: Increasing returns to scale, $e_p^h = 1.4$

The demand and revenue for Scenarios I, II, and III are illustrated in Fig. 3, Fig. 4, and Fig. 5, respectively. For each scenario, demand and revenue are plotted for five parking capacities. Before discussing the scenarios, let us redefine C_{rs}^i by substituting Eq. (1) and Eq. (2) into Eq. (3):

$$C_{rs}^i = \alpha t_{ri} + \beta \frac{l_i \mu_i k_i}{k_i - \sum_{(r,s)} d_{rs}^i h_{rs}^i} + (g_i + p_i h_{rs}^i) + \gamma w_{is} + \gamma w_{si} + \alpha t_{ir} \qquad \forall (r,s) \in V, \forall i \in \Omega(r,s) (35)$$

As is now shown in Eq. (35), h_{rs}^i generally influences C_{rs}^i in two separate terms (second and third terms of Eq. (35)). However, under Scenario I, given that $p_i h_{rs}^i = p_i 3 p_i^{-1} = 3$ is a constant, h_{rs}^i influences C_{rs}^i only via the second term. Hence, as p_i increases, h_{rs}^i decreases causing C_{rs}^i and consequently d_{rs} to approach their asymptotic values as is shown in Fig. 3. The revenue of this scenario also reaches its asymptotic value for the same reason. In Scenario II, demand initially increases with price and then it decreases as is shown in Fig. 4. The initial increase occurs because increasing p_i leads to a lower dwell time and lower generalized cost, which in turn increases demand as elaborated in Section 3. The latter decrease in demand occurs because p_i directly contributes to the generalized cost which reduces demand. The demand in Scenario III somewhat follows that same pattern as Scenario I (as shown in Remark 2 of Section 3) but the revenue patterns are different as shown in Fig. 5. In Scenario III, the revenues reach a peak value due to the higher influence of price on reducing dwell time. In all three scenarios, cases with higher parking capacities have higher demand, revenue, and occupancy due to the lower cost of searching for parking (second term of Eq. (35)). Moreover, for all parking capacities in all three scenarios, demand, revenue, and occupancy converge. The reason of convergence is that at high p_i values, dwell time and parking occupancy become so low that the parking capacity no longer imposes any restriction.

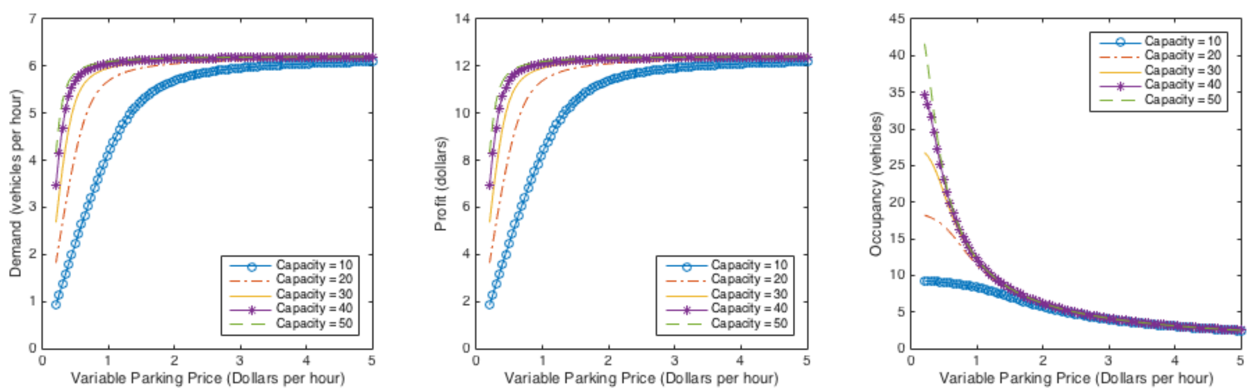

Fig. 3. Demand, revenue, and occupancy for Scenario I.

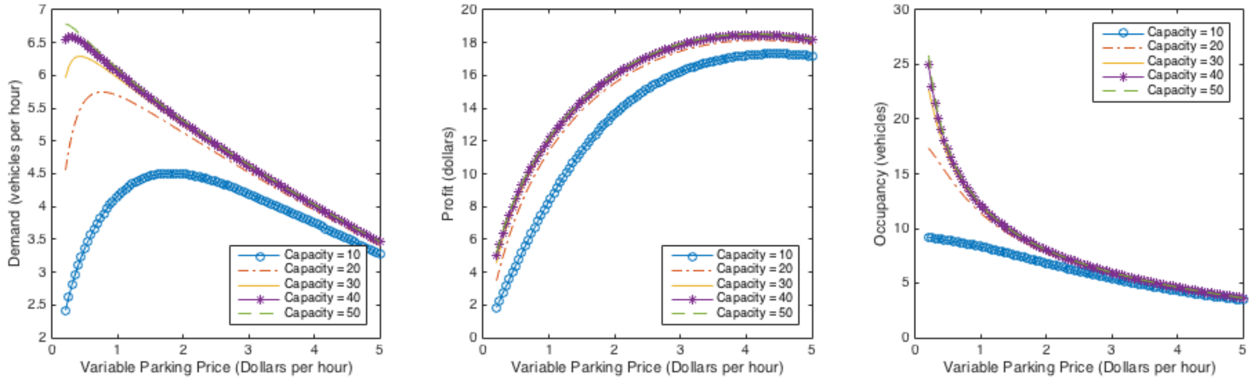

Fig. 4. Demand, revenue, and occupancy for Scenario II.

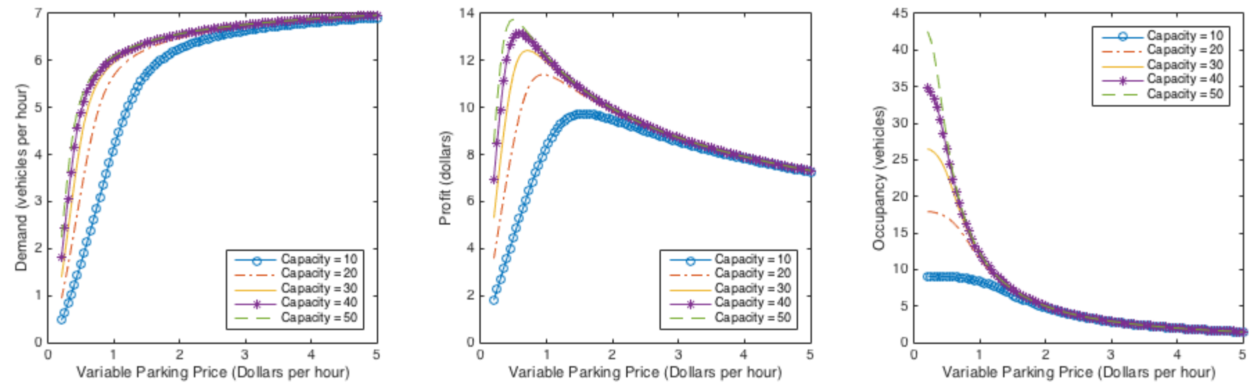

Fig. 5. Demand, revenue, and occupancy for Scenario III.

6.2 Second network: fixed demand and fixed parking capacity

The second network is a grid network 32 origins nodes, 49 destination nodes, and 64 parking areas as shown in Fig. 6. The network includes a total of 144 bidirectional traffic links and a total of 196 bidirectional walk paths that connect the parking areas to the final destination zones. Travel time on each walking link is fixed and equal to 5 minutes but the travel time of each traffic link is obtained from the BPR function $t = f[1 + (x/cap)^4]$ where $f = 5$ is the free-flow travel time and cap=1000 vehicles per hour is the capacity of each traffic link. The parking search time at each parking area is obtained from the BPR-type function $F(q) = \mu l[1 + (q/k)^3]$ in which $\mu = 0.5$ minutes $l = 1$ and $k = 100$ vehicles is the capacity each parking area. The dispersion parameter in the stochastic equilibrium model is set to $\theta = 0.9$. A demand of 1000 vehicles per hour is generated from each origin zone and evenly distributed between the 49 destination zones. The travelers of all origin-destination pairs are assumed to homogenous with a parking dwell time of 30 minutes which is obtained from Eq. (7) for a given variable parking price and a fixed parking price of zero.

The parking search time of each parking area is computed at equilibrium and presented in Fig. 6. It is depicted that the center of the network has a larger parking search time due to the higher availability of those parking areas to travelers. On the of the boundaries of the network, however, the parking search time is low because the parking areas serve only a limited number of final destinations. The error term for finding the equilibrium as shown in Eq. (31) is depicted in Fig. 7. Sensitivity analysis is performed on the dwell time and the results are presented in Fig. 9 in which the average parking search time and the total network travel time (including travel time and search time) are depicted. As illustrated, average parking search time increase with dwell time due to higher occupancy of the parking areas. Consequently, the longer search time increases the total network travel time as vehicles cruise to find a parking spot.

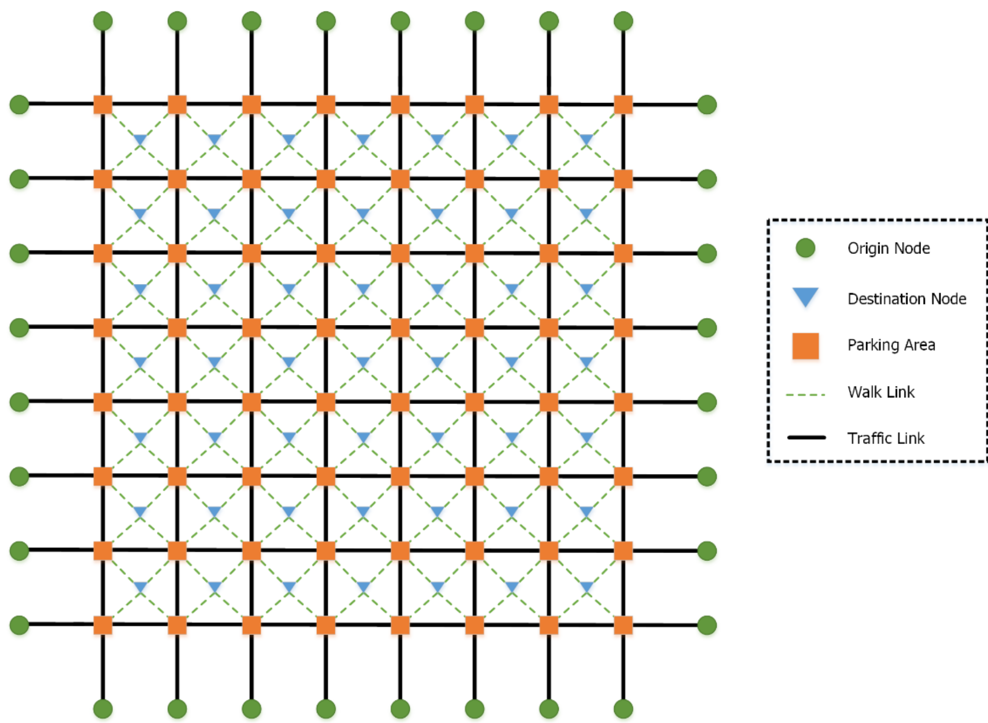

Fig. 6. Second example network.

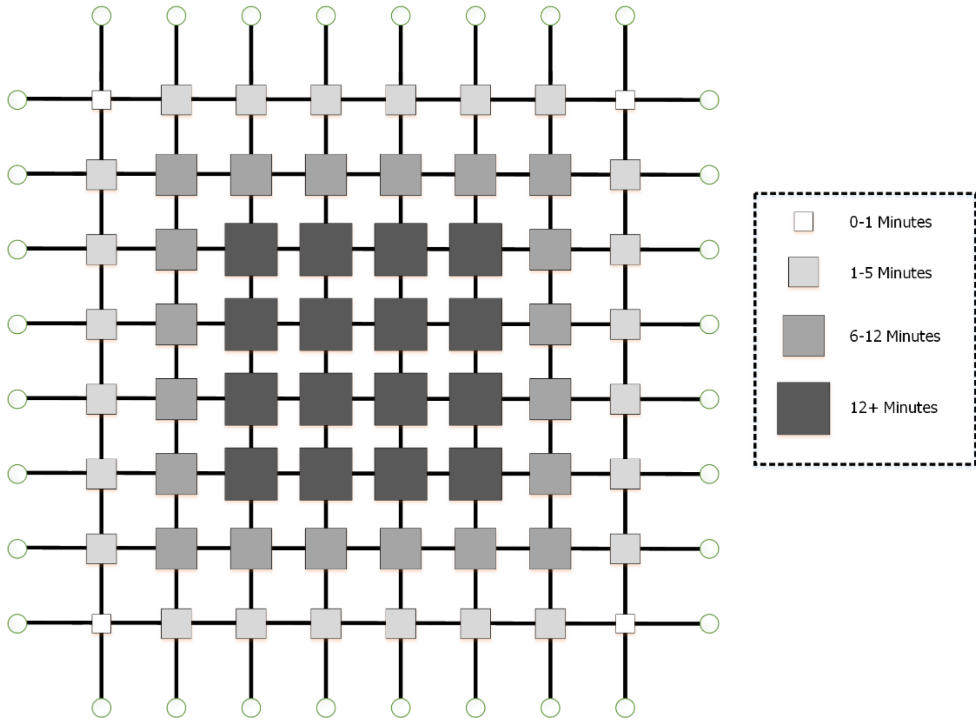

Fig. 7. Parking search time.

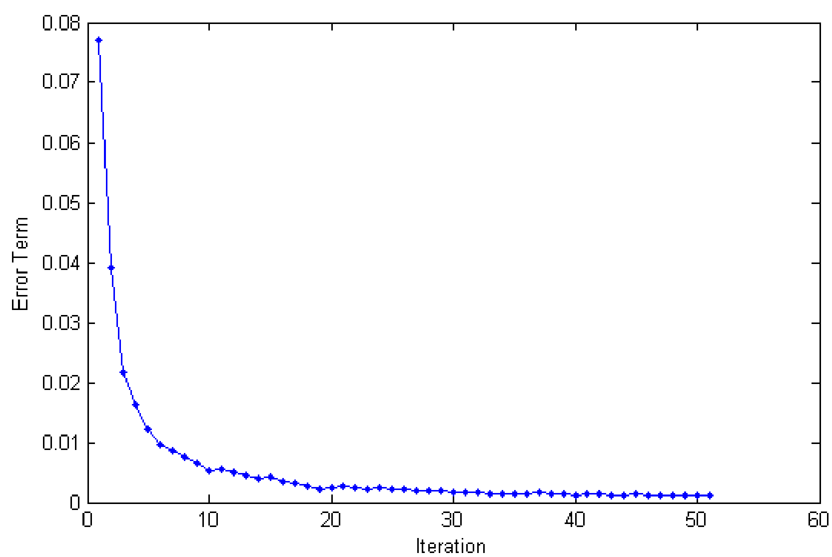

Fig. 8. Convergence of the algorithm.

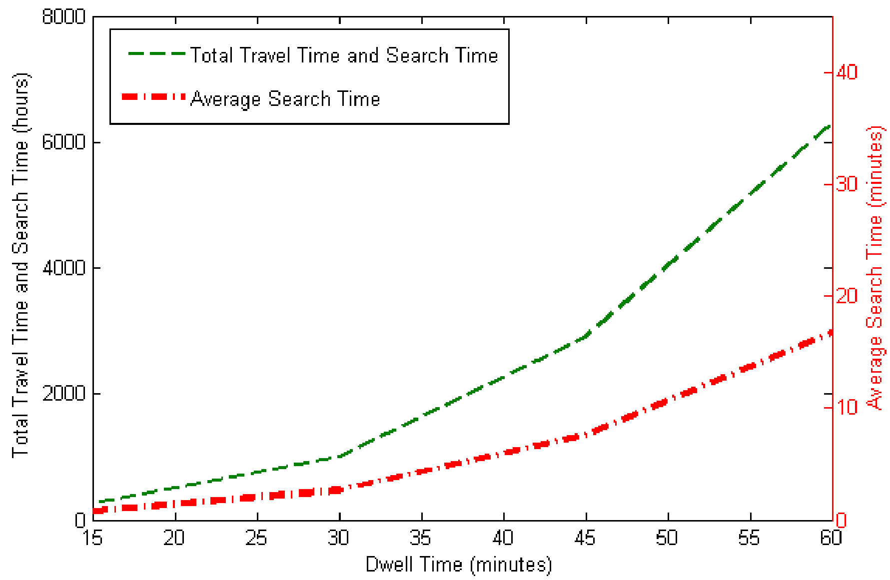

Fig. 9. Total parking search time and travel time, and average parking search time.

7 Conclusions

This paper investigates the impact of variable parking pricing on traffic conditions and parking search time. Parking pricing, if imposed wisely, has the potential to complement or even substitute road pricing. When imposed imprudently, however, variable parking pricing can increase the generated demand and created further congestion. This study shows that road pricing and parking fares are structurally different in how they influence the traffic equilibrium. While road pricing reduces demand, parking fares can reduce or induce the generated demand. To capture the emergent traffic equilibrium with parking, we present a Variational Inequality model and prove that it derives the equilibrium. Numerical experiments show that parking capacity is only influential in the equilibrium when the variable parking price is low. Analysis of a grid network depicts a larger parking search time at the center of the network with parking zones that are accessible more travelers.

Nomenclature

Sets

$G(N, A)$	Graph with node set N and arc set A
N	Set of nodes
R	Set of external nodes
I	Set of parking nodes
S	Set of internal zones

A	Set of arcs
A_d	Set of driving arcs
A_w	Set of walking arcs
V	Set of O-D pairs
$\Omega(r,s)$	Parking choice set of O-D pair $(r,s) \in V$ travelers
$\psi(r,s,i)$	Set of routes for O-D pair $(r,s) \in V$ travelers who choose parking $i \epsilon \Omega(r,s)$

Constants

w_b	Walking time on walking link $b \in A_w$
k_i	Capacity of parking $i \epsilon I$
$\Delta_{a,b}$	Link-path incidence matrix
σ_i	Maintenance cost of one spot at parking zone $i \in I$
l_i	Average searching time at parking $i \in I$
μ_i	Constant representing how drivers adopt occupancy information at parking $i \in I$
α	Marginal cost of each hour of driving time
β	Marginal cost of each hour of parking search time
γ	Marginal cost of each hour of walking time
θ	Dispersion parameter in the parking choice model

Decision variables

x_b	Flow on link $b \in A_d$
$\tau_b(x_b)$	Travel time on driving link $b \in A_d$
d_{rs}^i	Flow of O-D pair $(r,s) \in V$ travelers who choose parking $i \epsilon \Omega(r,s)$
d_{rs}	Flow of O-D pair $(r,s) \in V$ travelers
$d_{rs,a}^i$	Flow of O-D pair $(r,s) \in V$ who choose parking $i \epsilon \mathit{\Omega}(r,s)$ via route $a \epsilon \psi(r,s,i)$
d^i	Flow of travelers into parking $i \epsilon I$

q_i	Occupancy of parking $i \in I$
h_{rs}^i	Dwell time of O-D pair $(r,s) \in V$ travelers who choose parking $i \epsilon \Omega(r,s)$
π_{rs}^i	Probability that an O-D pair (r,s) traveler chooses parking $i \epsilon \Omega(r,s)$
η_{rs}	Expected perceived travel cost of O-D pair $(r,s) \in V$ travelers
$D_{rs}(\eta_{rs})$	Demand function of O-D pair $(r,s) \in V$ travelers
C_{rs}^i	Observed cost of O-D pair (r,s) travelers who choose parking $i \epsilon \Omega(r,s)$
ε_{rs}^i	Unobserved cost of O-D pair (r,s) travelers who choose parking $i \epsilon \Omega(r,s)$
g_i	Fixed price of parking at zone $i \in I$ measured in dollars
p_i	Variable price of parking at zone $i \in I$ measured in dollars per hour
Γ	Feasible region of the Variational Inequality program
u_{rs}^i	Lagrange multiplier associated with conservation of flow for O-D pair (r,s) travelers who choose parking $i \epsilon \Omega(r,s)$
λ_{rs}	Lagrange multiplier associated with conservation of flow for O-D pair (r,s)
δ_i	Lagrange multiplier associated with conservation of flow at each parking zone $i \in I$
$\varphi_{rs,a}^i$	Lagrange multiplier associated with conservation of flow for O-D pair (r,s) travelers who choose parking $i \epsilon \Omega(r,s)$ via route $a \in \psi(r,s,i)$

Functions

$H_{rs}(p_i)$	Dwell time function for O-D pair (r,s) at parking zone $i \epsilon \Omega(r,s)$
$F_i(q_i)$	Searching time at parking $i \in I$
PM	Profit maximization function
SS	Social surplus function

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