Transportation Research Part A 98 (2017) 28–45

Contents lists available at ScienceDirect

Transportation Research Part A

journal homepage: www.elsevier.com/locate/tra

Impact of hourly parking pricing on travel demand

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A R T I C L E I N F O

Article history:
Received 27 December 2015
Received in revised form 21 November 2016
Accepted 23 January 2017
Available online 23 February 2017

Keywords:
Parking
Pricing
Traffic assignment
Variational Inequality

A B S T R A C T

Efficient parking management strategies are vital in central business districts of cities where parking is limited and congestion is intense. Hourly parking pricing is a common parking management strategy where vehicles pay based on their parking duration (dwell time). In this paper, we derive comparative static effects for a small network to show that road pricing and hourly parking pricing are structurally different in how they influence the traffic equilibrium with elastic demand. Whereas road pricing strictly reduces demand, hourly parking pricing can reduce or induce demand depending on the parking dwell time elasticity (to the hourly parking price). When dwell time is elastic, demand always increases with parking price. However, when dwell time is inelastic, demand may increase or decrease with the parking price. Hence, hourly parking pricing can actually cause higher congestion and decay social welfare if imposed imprudently. For larger networks, we present a Variational Inequality model that characterizes the emergent equilibrium. Numerical experiments on a large network validate our analytical findings from a smaller and stylized case study. Our results also show a lower standard deviation in the parking search time (i.e., time to find a parking spot) when dwell time is highly elastic to the hourly parking price.

1. Introduction

1.1. Motivation

Efficient parking management strategies are vital in Central Business Districts (CBDs) of cities where parking is limited and congestion is intense. A great deal of parking demand in these regions is generated by travelers who visit their destination for some specified period (called dwell time) before returning to their origin location (Anderson and de Palma, 2007). To find parking for these activities (e.g., shopping activities), travelers incur a cost comprised of traveling to a chosen parking area, searching for a spot, paying the parking price, and walking to the final destination.

In day-to-day equilibrium conditions or in the presence of information systems such as mobile apps, travelers adjust their travel patterns to minimize their experienced costs. This adjustment includes choosing an affordable parking area in the vicinity of the final destination. Parking areas are underground or multi-floor parking garages, surface lots, or a collection of on-street parking spots. They can be public or private and generally require a parking fee with a fixed price (e.g. \$3 for entrance) and an hourly price (e.g. \$0.5 per hour). The hourly price plays a key role in parking management. Its impact on parking demand is twofold. First, increasing the hourly price of a parking facility increases user costs and explicitly

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http://dx.doi.org/10.1016/j.tra.2017.01.023
0965-8564/

Nomenclature

Sets

$G(N, A)$	graph with node set N and arc set A
N	set of nodes
R	set of external nodes
I	set of parking nodes
S	set of internal zones
A	set of arcs
A_d	set of driving arcs
A_w	set of walking arcs
V	set of O-D pairs
$\Omega(r, s)$	parking choice set of O-D pair $(r, s) \in V$ travelers
$\psi(r, s, i)$	set of routes for O-D pair $(r, s) \in V$ travelers who choose parking $i \in \Omega(r, s)$

Constants

w_b	walking time on walking link $b \in A_w$
k_i	capacity of parking $i \in I$
$\Delta_{a,b}$	link-path incidence matrix
σ_i	maintenance cost of one spot at parking zone $i \in I$
l_i	average searching time at parking $i \in I$
μ_i	constant representing how drivers adopt occupancy information at parking $i \in I$
α	marginal cost of each hour of driving time
β	marginal cost of each hour of parking search time
γ	marginal cost of each hour of walking time
θ	dispersion parameter in the parking choice model

Decision variables

x_b	flow on link $b \in A_d$
$\tau_b(x_b)$	travel time on driving link $b \in A_d$
d^i_{rs}	flow of O-D pair $(r, s) \in V$ travelers who choose parking $i \in \Omega(r, s)$
d_{rs}	flow of O-D pair $(r, s) \in V$ travelers
$d^i_{rs,a}$	flow of O-D pair $(r, s) \in V$ who choose parking $i \in \Omega(r, s)$ via route $a \in \psi(r, s, i)$
d^i	flow of travelers into parking $i \in I$
q_i	occupancy of parking $i \in I$
h^i_{rs}	dwell time of O-D pair $(r, s) \in V$ travelers who choose parking $i \in \Omega(r, s)$
π^i_{rs}	probability that an O-D pair (r, s) traveler chooses parking $i \in \Omega(r, s)$
η_{rs}	expected perceived travel cost of O-D pair $(r, s) \in V$ travelers
$D_{rs}(\eta_{rs})$	demand function of O-D pair $(r, s) \in V$ travelers
C^i_{rs}	observed cost of O-D pair (r, s) travelers who choose parking $i \in \Omega(r, s)$
ε^i_{rs}	unobserved cost of O-D pair (r, s) travelers who choose parking $i \in \Omega(r, s)$
p_i	hourly price of parking at zone $i \in I$ measured in dollars per hour
Γ	feasible region of the Variational Inequality program
u^i_{rs}	Lagrange multiplier associated with conservation of flow for O-D pair (r, s) travelers who choose parking $i \in \Omega(r, s)$
λ_{rs}	Lagrange multiplier associated with conservation of flow for O-D pair (r, s)
δ_i	Lagrange multiplier associated with conservation of flow at each parking zone $i \in I$
$\varphi^i_{rs,a}$	Lagrange multiplier associated with conservation of flow for O-D pair (r, s) travelers who choose parking $i \in \Omega(r, s)$ via route $a \in \psi(r, s, i)$

Functions

$H_{rs}(p_i)$	dwell time function for O-D pair (r, s) at parking zone $i \in \Omega(r, s)$
$F_i(q_i)$	searching time at parking $i \in I$
PM	profit maximization function
SS	social surplus function

reduces demand. Second, the same increase in the hourly price motivates travelers to shorten their dwell time which leads to lower parking occupancy and searching time (to find a spot), but higher demand. Hence, the hourly parking price influences travel demand in two counterbalancing ways. This paper investigates the impact of hourly parking pricing on traffic equilibrium conditions and parking search time. We show that, despite intuition, hourly parking pricing can actually increase demand if imposed imprudently.

Parking pricing has long been advocated and deployed as a policy to reduce congestion. Whereas a wealth of research is dedicated to parking pricing in areas such as mall parking services (Chang et al., 2014), commercial vehicle parking (Marcucci et al., 2015; Nourinejad et al., 2014; Nourinejad and Roorda, 2016; Amer and Chow, 2016), curbside parking (Millard-Ball et al., 2014), private firm parking (Tsai and Chu, 2006; Nourinejad and Roorda, 2014), morning commute parking (Qian et al., 2011; Liu et al., 2014a, 2014b; Liu and Geroliminis, 2016), dynamic parking pricing (Zheng and Geroliminis, 2016; Zakharenko, 2016), and parking permits (Rosenfield et al., 2016), fewer studies have investigated the role of hourly pricing as a travel demand management (TDM) policy. Among the few, Glazer and Niskanen (1992) showed that if roads are sub-optimally priced or not priced at all, then fixed parking pricing can increase welfare but hourly pricing may not. Glazer and Niskanen (1992) conclude that, despite intuition, an increase in the hourly parking price will induce demand because more parking spaces become available as drivers shorten their dwell times. In this paper, we show that this is not always the case by proving that the changes in demand (with respect to the hourly price) are sensitive to the elasticity of parking dwell time (with respect to the hourly price). Thereafter, we define three market regimes, investigate the equilibrium conditions, and present a network-based model to capture the spatial effects of hourly parking pricing.

1.2. Background

Parking studies are broadly categorized based on modeling framework, search mechanism, and turnover. The two main modeling frameworks are simulation and analytic formulations. Simulations capture complex dynamics of parking but require detailed data for calibration. Often, lack of sufficient data is accommodated by applying behavioral assumptions which are mostly inconsistent among different studies (Benenson et al., 2008; Gallo et al., 2011; Nourinejad et al., 2014). In Benenson et al. (2008), for instance, vehicles relinquish their on-street parking search after some time threshold (10 min) and head for off-street parking instead. In Nourinejad et al. (2014), on the other hand, vehicles start to search for parking when within 500 m of their final destination. In comparison, analytical models, with a few exceptions, are less data-hungry and more insightful but are generally aggregate and not amenable to detailed results (Arnott and Inci, 2006; Arnott and Rowse, 1999; Anderson and De Palma, 2007). In Arnott and Inci (2006), for instance, a parking model is developed for downtown areas with equal-sized blocks and a constant demand over the region. Although aggregate, the model provides very useful insights such as showing that it is efficient to raise the on-street parking fee to the point where cruising for parking is eliminated without parking becoming unsaturated. More recently, there is growing advocacy for network-based analytical and traffic assignment models that allow for a finer level of policy support. Boyles et al. (2014) and Qian and Rajagopal (2014) formulate equilibrium models to assign vehicles to spatially disaggregate parking areas.

Searching mechanisms are either zone-based[1] or link-based. In zone-based searching, vehicles only start searching for a spot when they reach a zone and each zone is associated with a search time which is assumed to be a function of the zone's occupancy (i.e., number of occupied parking spots) (Qian and Rajagopal, 2014). Applications of zone-based searching are not limited to parking. In taxi equilibrium models, taxi drivers search for passengers in different zones and incur a searching cost which is generally assumed to be a function of the total number of searching taxis and passengers in that zone. Taxi searching time is usually lower when there are more passengers and fewer taxis in the zone (Yang and Wong, 1998; Yang et al., 2002, 2010a, 2010b). In link-based searching, vehicles search for a spot in any of the links that are on their route to a final destination zone. One of the interesting implications of a link-based search model, as is shown in Boyles et al. (2014), is the smooth transition of vehicles from "driving" to "searching for parking" which is inherent in the equilibrium structure of the model.

Parking studies are classified into zero and non-zero turnover rate models. Turnover refers to the rate at which vehicles leave a parking area. Hence, zero turnover parking indicates that vehicles only enter parking areas without leaving. This type of parking is common in the morning commute context where the major concern is the dynamic arrival pattern of vehicles at the parking zones. These studies are usually defined for stylized settings such as a single bottleneck linear city (Zhang et al., 2008; Qian et al., 2012) or a parallel bottleneck city with several corridors (Zhang et al., 2011). A more general network-based zero turnover model is developed by Qian and Rajagopal (2014). Non-zero turnover models, on the other hand, are suitable for short duration activities such as shopping. These models capture both the arrival and departure rate of vehicles from each parking area. Under steady-state conditions, the arrival rate is equal to the departure rate from each parking area (Arnott, 2006, 2014; Arnott and Rowse, 2009, 2013; Arnott and Inci, 2010; Arnott et al., 2015). In non-zero turnover simulation models such as Guo et al. (2013) and Nourinejad et al. (2014), the sum of vehicles entering and leaving each parking area are assumed to be equal.

The policy implications of parking have also been the subject of many studies (Inci, 2014). Among the more innovative ones are parking permit schemes that involve distributing a fixed number of permits among travelers and restricting vehicles to spend the permits for parking (Zhang et al., 2011; Liu et al., 2014a, 2014b). He et al. (2015) study the optimal assignment

[1] By zone, we refer to either an off-street parking lot or a collection of on-street parking spaces.

of vehicles to parking spots while considering the competition game between the vehicles. They show the existence of multiple equilibria and propose a robust pricing scheme. Qian and Rajagopal (2014) study parking pricing strategies using real-time sensors to manage parking demand. Using parking pricing and information provision systems, Qian and Rajagopal (2014) propose a dynamic stabilized controller to minimize the total travel time in the system. Parking prices are then adjusted in real-time according to occupancy information collected from parking sensors. Finally, Xu et al. (2016) consider a policy where private parking slots can be shared between a pool of drivers.

1.3. Contributions and organization

In this paper, we present a non-zero turnover, zone-based search, analytical model for parking. Given the non-zero turnover rate, we consider both arrival and departure rates of vehicles to parking areas which are assumed to be equal under steady-state conditions. Our model is therefore distinguished from Qian and Rajagopal (2014) which is a zero turnover model. Contrary to Boyles et al. (2014), we use the zone-based search mechanism which, due to its simplicity, helps derivation of the analytical results and improves policy evaluation. The presented model is also distinguished from the analytical models of Arnott and Inci (2006), Arnott (2014), and Arnott et al. (2015) since it investigates parking patterns at a network level. Although our analytical findings are only derived for a simple case study, we show through numerical experimentation that our results are generalizable to larger networks as well.

We particularly focus on unassigned parking where drivers have to cruise to find a spot. These trips have shorter dwell times and tend to belong to frequent drivers. We present cases where parking supply can be varied such as in Arnott and Inci (2006) and cases where parking supply is fixed. Each parking zone has a specified capacity and can either be an off-street parking facility or a group of on-street parking spots. The modeled network is a CBD where travelers reside far away. This assumption is previously imposed by Anderson and De Palma (2007) as well.

The remainder of this paper is organized as follows. The model is presented in Section 2. Comparative statics effects of regulatory variables are investigated in Section 3. Equilibrium conditions are discussed in Section 4. Three market regimes are presented in Section 5. Numerical experiments are provided in Section 6. Conclusions are presented in Section 7.

2. The model

2.1. The network

Consider a transportation network $G(N, A)$ with node and arc sets N and A, respectively. To model parking, we further partition the node set N into external nodes denoted by R, parking zones denoted by I, and internal zones denoted by S so that $N = R \cup I \cup S$. Let $R = \{1, \ldots, r, \ldots, |R|\}$, $I = \{1, \ldots, i, \ldots, |I|\}$, and $S = \{1, \ldots, s, \ldots, |S|\}$. The reason for this terminology is that external zones are located at the boundary of a study region whereas internal zones are within the study region as shown in Fig. 1. External zones are gateways that provide accessibility to a region and internal zones are attraction locations (e.g. a shopping center) that vehicles want to visit.

Each vehicle completes two types of trips: inbound and outbound. In the inbound trip, vehicles leave an external zone, r, and drive to a parking zone, i. After parking, the inbound traveler walks from the parking zone, i, to an internal zone, s, as is shown in Fig. 1a. Hence, the path of every inbound traveler includes the sequence $r \rightarrow i \rightarrow s$. Outbound trips are the reverse direction of inbound trips. The path of every outbound vehicle includes the sequence $s \rightarrow i \rightarrow r$. Fig. 1a depicts the general inbound and outbound trip trajectories and Fig. 1b illustrates an example of internal and external zones where the internal zones are attraction locations in the Toronto CBD and the external zones represent the gateways to the CBD. Let us also partition the link Set A into A_d and A_w representing the driving and walking links, respectively, as is also shown in Fig. 1a.

The following sets are now defined. Let $V = R \times S$ be the set of external-internal zone pairs. For each pair (r, s), let $\Omega(r, s)$ be the set of parking zones that are within the parking zone choice-set of these travelers. The Set $\Omega(r, s)$ can be defined according to features such as walking distance from parking i to destination s and the cost of parking. Clearly, parking zones that are too far from the internal destinations zones are less likely to be included in the choice set. For every pair $(r, s) \in V$ and parking zone $i \in \Omega(r, s)$, let $\psi(r, s, i)$ be the set of routes for the segments of the tour that include driving links. Each route is comprised of a set of driving links connecting zone r to i and zone i to s. For instance, in Fig. 1a, there is only one route that includes the sequences of zones $r \rightarrow i \rightarrow s$.

Let $d^i_{rs,a}$ denote the flow of vehicles belonging to pair $(r, s) \in V$ that choose parking area $i \in \Omega(r, s)$ via route $a \in \psi(r, s, i)$. Let x_b be the flow and $\tau_b(x_b)$ the travel time of driving link $b \in A_d$, and let w_b be the walking time on link $b \in A_w$. It is commonly assumed that the travel time on driving link $b \in A_d$ is a continuous and monotonically increasing function of link flow x_b and the travel time on walking link $b \in A_w$ is independent of the flow. Let Δ represent the path-link incidence matrix where $\Delta_{a,b} = 1$ if link $b \in A_d$ is included in route $a \in \psi(r, s, i)$ and $\Delta_{a,b} = 0$, otherwise. Hence we have $x_b = \sum_{(r,s) \in V} \sum_{i \in \Omega(r,s)} \sum_{a \in \psi(r,s,i)} d^i_{rs,a} \Delta_{a,b}$.

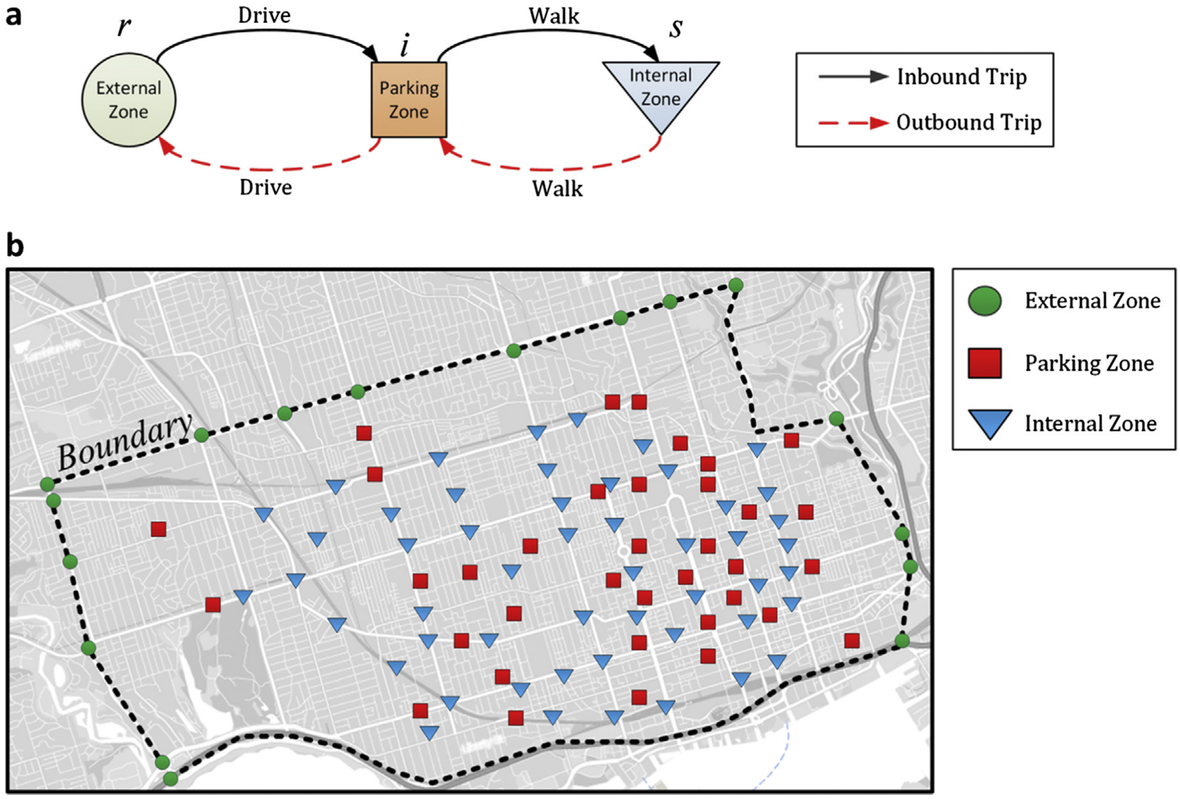

Fig. 1. (a) Inbound and outbound trip trajectories; (b) example of zone types.

2.2. The non-zero turnover parking process

The parking search process is explained in this section. First, the following assumption is imposed:

Assumption 1. Under equilibrium conditions, travelers park at a zone with the lowest generalized cost.

Assumption 1 is justified under at least two conditions. First, if the trips are recurrently performed, travelers become familiar with the process and choose to park at a zone with the lowest generalized cost. Second, when parking information such as parking occupancy is provided to users via apparatus such as mobile apps, travelers are better informed about which zone to choose for parking. Assumption 1 implies that travelers will not hop between parking zones (i.e. they will not drive from one parking area to another) and will instead choose the one with the lowest generalized cost. The cost of parking is comprised of the cost of traveling from the external zone to a parking zone, the cost of searching for parking, the parking fee which can include both a fixed and an hourly component, the cost of walking from the parking area to the internal zone, the cost of walking from the internal zone to the parking area, and the cost of driving from the parking area to the external zone.

Using Assumption 1, we can now analyze the parking pattern of travelers. Let $d^i_{rs}, \forall (r,s) \in V, i \in \Omega(r,s)$, be the flow of vehicles that originate at zone r, terminate at zone s, and park at zone i, and let $d_{rs} = \sum_{i\in\Omega(r,s)} d^i_{rs}$ be the total flow from r to s. All travelers of pair (r,s) that park at i, remain there for h^i_{rs} [hours] called the dwell time. This assumption is justified as travelers belonging to the same origin-destination pair are likely to be homogenous (Yang and Huang, 2005).

Let q_i be the total occupancy of parking $i \in I$ under equilibrium and let k_i be the capacity of parking i measured in vehicles. Note that k_i is a given whereas q_i is obtained from the equilibrium:

$$q_i = \sum_{(r,s)} d^i_{rs} h^i_{rs} \quad \forall i \in I \tag{1}$$

Parking search time is typically assumed to be a convex function of parking occupancy q_i and capacity k_i (Axhausen et al., 1994; Anderson and De Palma, 2007; Qian and Rajagopal, 2014). The general form of this function $F_i(q_i)$, as explained in Axhausen et al. (1994), is:

$$F_i(q_i) = \frac{l_i \mu_i}{1 - \frac{q_i}{k_i}} \quad \forall i \in I \tag{2}$$

where l_i is the average searching time in parking area i when occupancy is low or medium and μ_i is a constant representing how drivers react to occupancy information. When $\mu_i = 0$, drivers are unaware of the searching time and when $\mu_i = 1$ drivers are completely aware of searching time. Axhausen et al. (1994) estimated the search time function with a coefficient of determination $R^2 = 0.91$ for Frankfurt, Germany. The searching time function $F_i(q_i)$ asymptotically goes to infinity as q_i approaches k_i, i.e., $lim_{q_i \to k_i} F_i(q_i) = \infty$. This implies that a driver entering a full occupancy parking area will never find a spot.

2.3. Generalized travel costs

The hourly price of parking at $i \in I$ is p_i [dollars per hour]. Hence, for the pair (r, s) a traveler who chooses parking i pays $p_i h^i_{rs}$ dollars for parking. We can now derive the generalized travel costs. Let $C^i_{rs,a}$ be the generalized travel cost for travelers of pair (r, s) who choose parking $i \in \Omega(r, s)$ via route $a \in \psi(r, s, i)$. This cost is composed of the following six terms: (i) traveling from external zone r to parking i via route a with a travel time $t_{ri,a}$, (ii) searching for parking for a period of $F_i(q_i)$, (iii) a parking cost of $p_i h^i_{rs}$ dollars, (iv) walking from parking i to zone s, (v) walking from zone s to parking i, and (vi) traveling from parking i to external zone r via route a:

$$C^i_{rs,a} = \alpha t_{ri,a} + \beta F_i(q_i) + p_i h^i_{rs} + \gamma w_{is} + \gamma w_{si} + \alpha t_{ir,a} \quad \forall (r, s) \in W, \forall i \in \Omega(r, s), \forall a \in \psi(r, s, i) \tag{3}$$

In Eq. (3), α, β, and γ are the marginal cost of travel time, parking search time, and walking time, respectively. For the first term on the right-side of Eq. (3), we have $t_{ri,a} = \sum_b \tau_b \Delta_{a,b}$.

The minimum cost of the shortest route for a traveler of pair $(r, s) \in V$ that parks at zone $i \in \Omega(r, s)$ is $C^i_{rs} = \min_{a \in \psi(r,s,i)} C^i_{rs,a}$. However, C^i_{rs} only represents the observed cost of travel. Let us also assume an additional unobserved cost of ε^i_{rs} which is independently and identically Gumbel distributed for all parking zones $i \in I$ that can be chosen by travelers of pair (r, s). With this assumption, the probability that a pair $(r, s) \in V$ traveler chooses parking $i \in \Omega(r, s)$ is denoted by π^i_{rs} which is obtained from the following logit-based probability function:

$$\pi^i_{rs} = \frac{\exp(-\theta C^i_{rs})}{\sum_{j \in \Omega(r,s)} \exp(-\theta C^j_{rs})} \quad \forall i \in \Omega(r, s) \tag{4}$$

where θ is a dispersion parameter representing the variation in the cost perception of travelers. Eq. (4) relies on the following assumption:

Assumption 2. Travelers are stochastic in choosing a parking area but deterministic in choosing routes. This assumption is justified due to the availability and accuracy of route-guidance advanced traveler information systems.

We also assume that travel demand is a continuous and decreasing function of the generalized travel cost. The demand function is denoted by $D_{rs}(.)$ and the expected, generalized travel cost is denoted by η_{rs} for each $(r, s) \in V$. Hence, we have:

$$d_{rs} = D_{rs}(\eta_{rs}) \quad \forall (r, s) \in W \tag{5}$$

Given the logit-based parking choice model in Eq. (4), the expected minimum cost for each $(r, s) \in V$ is:

$$\eta_{rs} = E\left(\min_{i \in \Omega(r,s)} \{C^i_{rs}\}\right) = -\frac{1}{\theta} \ln\left(\sum_{i \in \Omega(r,s)} \exp(-\theta C^i_{rs})\right) \quad \forall (r, s) \in W \tag{6}$$

2.4. Parking dwell time

Recall that parking dwell time h^i_{rs} is the time spent by (r, s) travelers at parking zone $i \in \Omega(r, s)$. The following assumption is now imposed:

Assumption 3. The dwell time of pair (r, s) travelers at parking zone $i \in \Omega(r, s)$ is assumed to be a function of the hourly parking cost p_i. As p_i increases, dwell time decreases.

Let $H_{rs}(p_i)$ denote this function which is assumed to be convex and monotonically decreasing with p_i. It is also sound to assume that dwell time approaches zero as p_i tends to infinity, i.e. $\lim_{p_i \to \infty} H_{rs}(p_i) = 0$. The dwell time function is:

$$h^i_{rs} = H_{rs}(p_i) \quad \forall (r, s) \in W, \forall i \in \Omega(r, s) \tag{7}$$

We now investigate the impact of the hourly price, p_i, on the out-of-pocket cost of parking, $p_i h^i_{rs}$. The hourly parking price, p_i, may or may not increase $p_i h^i_{rs}$. If h^i_{rs} decreases slowly with p_i, then the term $p_i h^i_{rs}$ increases with p_i, thus showing that the travelers pay more when the hourly parking price is increased. On the other hand, if h^i_{rs} decreases rapidly with p_i, then travelers pay less when p_i is increased.

3. Comparative static effects of parking pricing

We use comparative static effects (De la Fuente, 2000) to show that road pricing and parking fares are structurally different in how they influence the traffic equilibrium. Whereas road pricing reduces demand, hourly parking pricing may reduce or induce demand. Mathematically, we have $\frac{dD}{d\hat{p}} < 0$ where $\hat{p}$ is the road toll, whereas $\frac{dD}{dp} > 0$ or $\frac{dD}{dp} < 0$ where p is the hourly parking price and D is the demand function. Consider the network of Fig. 1a with one origin r, one destination s, and one parking area i. A road toll $\hat{p}$ is imposed on the driving link (r, i) and an hourly parking price p is imposed on parking area i. The demand function is defined such that demand decreases with generalized cost, i.e., $\frac{dD(\eta)}{d\eta} < 0$. For the remainder of this section, we drop the subscripts r, i, and s for brevity.

The following two lemmas demonstrate the changes in demand with respect to the road toll, $\hat{p}$, and the hourly parking price, p.

Lemma 1. *Demand strictly decreases with the road toll, i.e.,* $\frac{dD}{d\hat{p}} < 0$.

Proof. Let us rewrite $\frac{dD}{d\hat{p}}$ as

$$\frac{dD}{d\hat{p}} = \frac{dD}{d\eta}\frac{d\eta}{d\hat{p}}. \tag{8}$$

It is already assumed that $\frac{dD}{d\eta} < 0$ as demand decreases with the generalized cost. It is also evident that $\frac{d\eta}{d\hat{p}} > 0$ because $\hat{p}$ is the out-of-pocket money paid by travelers to traverse road (r, i). Hence, the product of the two terms on the RHS of Eq. (8) is negative and $\frac{dD}{d\hat{p}} < 0$. □

Lemma 2. *Changing the hourly parking price, p, may induce or reduce demand, i.e.,* $\frac{dD}{dp} <> 0$.

Proof. Let us rewrite $\frac{dD}{dp}$ as

$$\frac{dD}{dp} = \frac{dD}{d\eta}\frac{d\eta}{dp}. \tag{9}$$

It is already assumed that $\frac{dD}{d\eta} < 0$ as demand decreases with the generalized cost. Hence, we focus on the second term on the RHS of Eq. (9). By taking the derivative, $d\eta/dp$, we have

$$\frac{d\eta}{dp} = \frac{d(Hp)}{dp} + \frac{dF}{dp}. \tag{10}$$

By taking the derivative, dF/dp, the second term on the RHS of Eq. (10) can be rewritten as

$$\frac{dF}{dp} = \frac{\mu l[(dH/dp)D + (dD/dp)H]}{k(1 - Hd/k)^2}. \tag{11}$$

By inputting Eq. (11) into Eq. (10), inputting Eq. (10) into Eq. (9), and simplifying the terms, we have

$$\frac{dD}{dp} = \frac{dD}{d\eta}\left[\frac{(d(Hp)/dp) + \omega(dH/dp)D}{1 - \omega H(dD/d\eta)}\right] \tag{12}$$

where $\omega = \mu/[k(1 - HD/k)^2] > 0$. Analysis of Eq. (12) concludes the following:

$$\frac{dD}{dp} > 0 \quad \text{if } D > D^* \tag{13a}$$

$$\frac{dD}{dp} < 0 \quad \text{if } D < D^* \tag{13b}$$

where $D^* = \frac{-(dHp/dp)}{\omega(dH/dp)}$. Eq. (13) shows that marginal change of demand with respect to the hourly parking price, p, depends on the value of the materialized demand, D. If $D > D^*$, then travel demand, D, increases with the hourly parking price, p, and if $D < D^*$, then travel demand, D, decreases with the hourly parking price, p. □

Lemma 1 and 2 show that although road pricing reduces demand, variable parking pricing can reduce or induce demand. This implies that parking pricing is not always an appropriate policy for lowering congestion. Lemma 2 also has the following remark:

Remark 1. The hourly parking price has the same effect as the road toll when travelers' dwell time is insensitive to the hourly parking price.

Proof. When traveler dwell time is insensitive to the hourly parking cost (i.e. $\mathrm{d}H/\mathrm{d}p \to 0$), we have $D^* = \frac{-H}{\omega(\mathrm{d}H/\mathrm{d}p)} \to \infty$ which, according to Eq. (13b) indicates, that demand, D, strictly decreases with hourly parking price, p. In other words, when $\mathrm{d}H/\mathrm{d}p \to 0$, the hourly parking price has a similar impact on demand as a road toll. □

Let $e_p^h = \frac{\partial H}{\partial p}\frac{p}{H}$ be the elasticity of dwell time with respect to hourly parking price. By definition, dwell time is elastic when $e_p^h \leqslant -1$ and inelastic when $-1 < e_p^h \leqslant 0$. We now summarize the findings of this section in the following proposition:

Proposition 1. *Demand increases with hourly parking price (i.e.,* $\frac{\mathrm{d}D}{\mathrm{d}p} > 0$*) when dwell time is elastic (i.e.,* $e_p^h \leqslant -1$*). However, when dwell time is inelastic (i.e.,* $-1 < e_p^h \leqslant 0$*), demand may increase or decrease with hourly parking price (i.e.,* $\frac{\mathrm{d}D}{\mathrm{d}p} <> 0$*).*

Proof. Given that $e_p^h = \frac{\partial H}{\partial p}\frac{p}{H}$ and $\frac{\partial(pH)}{\partial p} = (1 + e_p^h)H$, we can rewrite D^* in Eq. (13) as $D^* = -p(1 + e_p^h)/\omega e_p^h$. When dwell time is inelastic (i.e., $-1 < e_p^h \leqslant 0$) we have $D^* > 0$ indicating that the demand both increases and decreases with the hourly parking price, p. However, when dwell time is elastic (i.e., $e_p^h \leqslant -1$), we have $D^* \leqslant 0$ which, according to Eq. (13), indicates that demand strictly increases with hourly parking price, p. □

4. Equilibrium conditions

4.1. A Variational Inequality formulation

In this section, we formulate the equilibrium problem using Variational Inequality (VI). The idea behind VI is to find an equilibrium point (say vector d^*) within a feasible (closed and compact) solution space such that for all other points (say d) in the solution space, we have $Z(d^*)^T(d - d^*) \geqslant 0$, where Z is a continuous function. Informally, d^* is a point in the feasible region that does not bear any force from the function Z. For novel applications of VI, see Wong et al. (2008) which presents a taxi equilibrium model and Nourinejad et al. (2016) which uses VI to model activity patterns in the presence of vehicle-to-grid technology.

Let us define the feasible region Γ route flows of the equilibrium model as the following set of equations where d^i is the flow of vehicles to parking zone $i \in I$ and the variables in brackets are the dual variables.

$$\sum_{a\in\psi(r,s,i)} d_{rs,a}^i = d_{rs}^i \quad [u_{rs}^i] \quad \forall (r,s) \in V, \forall i \in \Omega(r,s) \tag{14a}$$

$$\sum_{i\in\Omega(r,s)} d_{rs}^i = d_{rs} \quad [\lambda_{rs}] \quad \forall (r,s) \in V \tag{14b}$$

$$d^i = \sum_{(r,s)\in V} d_{rs}^i \quad [\delta_i] \quad \forall i \in I \tag{14c}$$

$$d_{rs,a}^i \geqslant 0 \quad [\varphi_{rs,a}^i] \quad \forall (r,s) \in V, \forall i \in \Omega(r,s) \tag{14d}$$

Constraints (14a) and (14b) ensure conservation of flow, constraints (14c) represent occupancy of parking i, and constraints (14d) ensure non-negativity of path flows.

For clarity, let us now partition the cost $C_{rs,a}^i$ (as shown in Eq. (3)) into the following terms:

$$C_{rs,a}^i = \varsigma_{rs,a}^i + \beta F_i(q_i) \quad \forall (r,s) \in V, \forall i \in \Omega(r,s), \forall a \in \psi(r,s,i) \tag{15}$$

where $\varsigma_{rs,a}^i = \alpha t_{ri,a} + \gamma w_{is} + \gamma w_{si} + \alpha t_{ir,a} + (g_i + p_i h_{rs}^i)$ represents the total observed travel cost including the cost of driving from r to i, walking from i to s, walking from s to i, parking at parking area i, and driving from i to r. The VI model is now presented as follows. Let $\boldsymbol{d} = \{d_{rs,a}^i \in \Gamma\}$ be a feasible solution. We plan to find the equilibrium solution $\boldsymbol{d}^* = \{d_{rs,a}^{i^*} \in \Gamma\}$ by showing that it always satisfies the following inequality:

$$\sum_{(r,s)\in V}\left(\sum_{i\in\Omega(r,s)}\left(\sum_{a\in\psi(r,s,i)}\varsigma^{i}_{rs,a}(\boldsymbol{d}^{*})(d^{i}_{rs,a}-d^{i^{*}}_{rs,a})+\frac{1}{\theta}\ln d^{i^{*}}_{rs}(d^{i}_{rs}-d^{i^{*}}_{rs})\right)-\frac{1}{\theta}\ln d^{*}_{rs}(d_{rs}-d^{*}_{rs})-D^{-1}_{rs}(d^{*}_{rs})(d_{rs}-d^{*}_{rs})\right)$$
$$+\beta\sum_{i}F_{i}(q^{*}_{i})(d^{i}-d^{i^{*}})\geqslant 0\quad\forall\boldsymbol{d}\in\Gamma \tag{16}$$

The Karush-Kuhn-Tucker (KKT) conditions of the VI program in Eq. (16) are derived as

$$d^{i}_{rs,a}:\ varsigma^{i}_{rs,a}(\boldsymbol{d}^{*})-u^{i}_{rs}-\varphi^{i}_{rs,a}=0\quad\forall(r,s)\in V,\forall i\in\Omega(r,s) \tag{17}$$

$$d^{i}_{rs}:\ u^{i}_{rs}+\delta_{i}-\lambda_{rs}+\frac{1}{\theta}\ln d^{i}_{rs}=0\qquad\forall(r,s)\in V,\forall i\in\Omega(r,s) \tag{18}$$

$$d_{rs}:\ \lambda_{rs}-D^{-1}_{rs}(d_{rs})-\frac{1}{\theta}\ln d_{rs}=0\quad\forall(r,s)\in V \tag{19}$$

$$d^{i}:\ \beta F_{i}(q_{i})-\delta_{i}=0\quad\forall i\in I \tag{20}$$

The complementarity conditions include constraints (14a)–(14d) and the following two:

$$d^{i}_{rs,a}\varphi^{i}_{rs,a}=0\quad\forall(r,s)\in V,\forall i\in\Omega(r,s),\forall a\in\psi(r,s,i) \tag{21}$$

$$\varphi^{i}_{rs,a}\geqslant 0\quad\forall(r,s)\in V,\forall i\in\Omega(r,s),\forall a\in\psi(r,s,i) \tag{22}$$

At equilibrium, δ_i is interpreted as the cost of searching at parking area i as per Eq. (29) and u^i_{rs} is interpreted as the minimum generalized travel cost (both driving and walking) of pair $(r,s)\in W$ travelers parking at zone i as per (Eq. (17)). We now show that the presented VI in equivalent to the equilibrium conditions of Section 2.

First, assume that demand is always non-negative $d^i_{rs,a}>0$, so that $\varphi^i_{rs,a}=0$. Given that $\varphi^i_{rs,a}=0$, taking the exponential function of both sides of Eq. (18) and simplifying the terms, we have

$$d^{i}_{rs}=\exp(-\theta(u^{i}_{rs}+\delta_{i}-\lambda_{rs}))\quad\forall(r,s)\in V,\forall i\in\Omega(r,s) \tag{23}$$

Using Eq. (14b), (Eq. (23)) can be rewritten as:

$$\sum_{i}d^{i}_{rs}=\exp(\theta\lambda_{rs})\sum_{i}\exp(-\theta(u^{i}_{rs}+\delta_{i}))=d_{rs}\quad\forall(r,s)\in V \tag{24}$$

Thus,

$$\exp(\theta\lambda_{rs})=\frac{d_{rs}}{\sum_{i}\exp(-\theta(u^{i}_{rs}+\delta_{i}))}\qquad\forall(r,s)\in V \tag{25}$$

Substituting Eq. (25) into Eq. (23) gives

$$d^{i}_{rs}=\frac{\exp(-\theta(u^{i}_{rs}+\delta_{i})}{\sum_{j}\exp(-\theta(u^{j}_{rs}+\delta_{j}))}d_{rs}\quad\forall(r,s)\in V,\forall i\in\Omega(r,s) \tag{26}$$

where the term δ_i can be related to the cost of searching at parking area i. This makes Eq. (26) equivalent to the logit-based choice probability indicating that $d^i_{rs}=\pi^i_{rs}d_{rs}$.

Eq. (19) can also be reorganized as

$$\lambda_{rs}=\frac{1}{\theta}\ln d_{rs}-\frac{1}{\theta}\ln\sum_{i}\exp(-\theta(u^{i}_{rs}+\delta_{i}))\qquad\forall(r,s)\in V \tag{27}$$

Substituting Eq. (27) into Eq. (25) gives:

$$D^{-1}_{rs}(d_{rs})=-\frac{1}{\theta}\ln\sum_{i}\exp(-\theta(u^{i}_{rs}+\delta_{i}))\quad\forall(r,s)\in V \tag{28}$$

which is equivalent to Eq. (6) as the demand function.

We have shown that the solution of the VI program satisfies all the functional relationships that are required by the parking model as defined in Section 2. The VI program has at least one solution when its feasible region, Γ, is a compact and convex set. Given that the feasible region Γ as is a set of linear constraints, and given that the VI function in Eq. (16) is continuous within the feasible region, we conclude that the VI has at least one solution (Florian et al., 2002).

4.2. Solving for equilibrium

An extensive review of solution algorithms for finding the traffic equilibrium is presented by Patriksson (1994). To solve the VI, traffic flows are assigned to parking areas $(d^i, \forall i)$ to find parking search times. Calculating parking search times can lead to infeasible solutions when the parking occupancy is larger than the parking capacity, i.e. $q_i \geqslant k_i$, because the search time function (Eq. (2)) is discontinuous with a vertical asymptote. To rectify this issue, the parking search time function is replaced with the following BPR-type equation:

$$F_i(q_i) = l_i\mu_i\left[1 + \left(\frac{q_i}{k_i}\right)^{\vartheta}\right] \tag{29}$$

where l_i is the average searching time in parking area i, μ_i is a constant representing how drivers adopt occupancy information, and ϑ is a calibration parameter. The parking search times are then used to find generalized costs and the origin-destination demands. The algorithm terminates upon convergence. The steps of the algorithm are the following:

Step 1. Initialization

Set the iteration number $\upsilon = 0$. Select an initial feasible demand solution $\boldsymbol{d}^{\upsilon}$. The feasible solution can be obtained by setting all travel times equal to free-flow travel times and setting the parking search time equal to zero for all parking areas.

Step 2. Computation of generalized costs

First, using $\boldsymbol{d}^{\upsilon}$, find the flow of vehicles into each parking area. The product of vehicle flows into each parking area and the parking dwell times (obtained for a given hourly price) gives parking occupancy which is input to Eq. (29) to find the search time of each parking area. Second, using $\boldsymbol{d}^{\upsilon}$, find the travel times and the generalized costs as per Eq. (3).

Step 3. Direction finding

Perform a stochastic network loading procedure on the current set of link travel times. This yields an auxiliary link flow vector $\widehat{\boldsymbol{d}}$.

Step 4. Method of successive averages

Using the demand obtained from Step 3, find the new flow pattern by setting

$$\boldsymbol{d}^{\upsilon+1} = \frac{\upsilon - 1}{\upsilon}\boldsymbol{d}^{\upsilon} + \frac{1}{\upsilon}\widehat{\boldsymbol{d}} \tag{30}$$

Step 5. Convergence test

Terminate if the following condition is satisfied with $\varkappa$ being a small number. Otherwise, set $\upsilon \to \upsilon + 1$ and go to Step 2.

$$\frac{\sqrt{\sum\left(\boldsymbol{d}^{\upsilon+1} - \boldsymbol{d}^{\upsilon}\right)^2}}{\sum \boldsymbol{d}^{\upsilon}} \leqslant \varkappa \tag{31}$$

5. Market regimes

Let us first assume that a single operator is in charge of managing all the parking facilities. This operator can be either a public or a private entity. In such cases, the two objectives of interest are profit maximization (denoted by *PM*) and social surplus maximization (denoted by *SS*). The former can be associated to the private and the latter to public authorities. The parking profit, *PM*, is:

$$PM = \sum_{(r,s)\in V}\sum_{i\in\Omega(r,s)}[(p_i h^i_{rs} + g_i)d^i_{rs}] - \sum_{i\in I} k_i\sigma_i \tag{32}$$

where the first term is the revenue from parking and the second term is the maintenance cost of all parking spots with σ_i denoting the maintenance cost of one parking spot at parking zone $i \in I$. The maintenance cost is not necessarily the cost of physical rehabilitation and can include other supervisory costs such as the cost of parking enforcement for on-street parking. The second objective function is social surplus which is:

$$SS = \sum_{(r,s)\in V}\int_0^{d_{rs}} D^{-1}_{rs}(z)dz - \sum_{i\in I} k_i\sigma_i \tag{33}$$

where $D^{-1}_{rs}(z)$ is the inverse of the demand function. With the two objective functions, we can now define the following three markets: (i) monopoly, (ii) first best, and (iii) second best. Let us assume for now that the parking operator has monopoly rights and can simultaneously decide on the capacity and the fee structure of all parking zones. Under this market, the objective is to maximize the total profit as shown in Eq. (32). Alternatively, in the first-best market, the objective is to maximize social surplus and in the second-best market, the objective is to maximize social welfare while ensuring a positive profit. Hence, under the second-best market we have:

$$\text{maximize} \quad \sum_{(r,s)\in V} \int_0^{d_{rs}} D_{rs}^{-1}(z)dz - \sum_{i\in I} k_i\sigma_i$$

subject to

$$\sum_{(r,s)\in V} \sum_{i\in\Omega(r,s)} [(p_i h_{rs}^i + g_i) d_{rs}^i] \geqslant \sum_{i\in I} k_i\sigma_i \tag{34}$$

6. Numerical experiments: the case of the city of Toronto

The first set of numerical experiments are performed on a simple case study with variable parking capacity. Thereafter, we present a network with fixed parking capacity and show that the analytical remarks are generalizable to larger networks.

6.1. A simple example with one origin, one destination, and one parking area

We present a simple example to visually present the three defined market regimes of Section 5. Consider the network in Fig. 1a with one O-D pair (r, s) and one parking zone $i \in \Omega(r, s)$. Let $\alpha = \beta = 10$ dollars per hour, $w_{si} = w_{is} = 0$ h, $\gamma = 0$ dollars per hour, $\theta = 1$, $\mu_i = 1$, and $l_i = 3$ min. The functions are defined as follows. Let $t_{ri} = t_{ir} = 0.5 + \frac{x^2}{1000}$ [measured in hours] where x is the total demand obtained from the demand function $x = D_{rs}(\eta_{rs}) = 20 - \eta_{rs}$. The dwell time function is $H_{rs}(p_i) = 3p_i^{-0.4}$.

The profit and social surplus contours are depicted in Fig. 2 for the simple example. As illustrated, the monopoly equilibrium occurs at the optimum of the profit objective function and the first-best equilibrium occurs at the optimum of the social surplus contours. The second-best equilibrium has to lie on the zero profit line where social surplus is maximized.

We further investigate the generated profits from the following three dwell time function scenarios:

I. $H_{rs}(p_i) = 3p_i^{-1}$: Unit-elastic, $e_p^h = -1$
II. $H_{rs}(p_i) = 3p_i^{-0.4}$: Inelastic, $e_p^h = -0.4$
III. $H_{rs}(p_i) = 3p_i^{-1.4}$: Elastic, $e_p^h = -1.4$

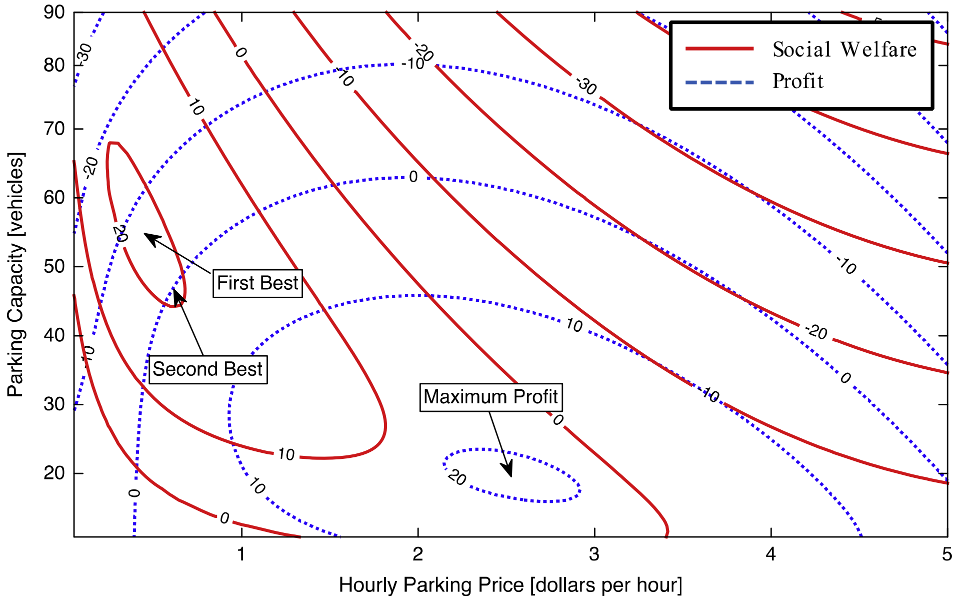

Fig. 2. Profit and social surplus contours for the simple example.

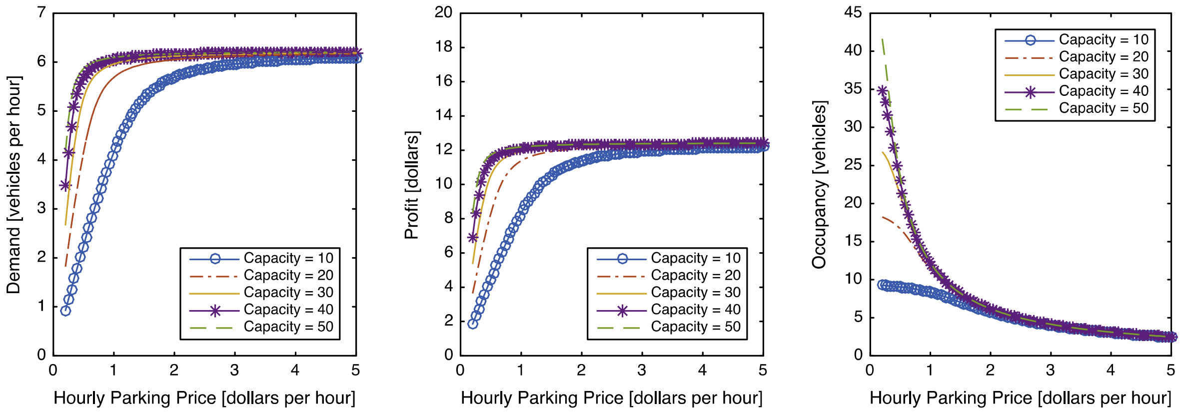

Fig. 3. Demand, profit, and occupancy for Scenario I with unit elasticity dwell time.

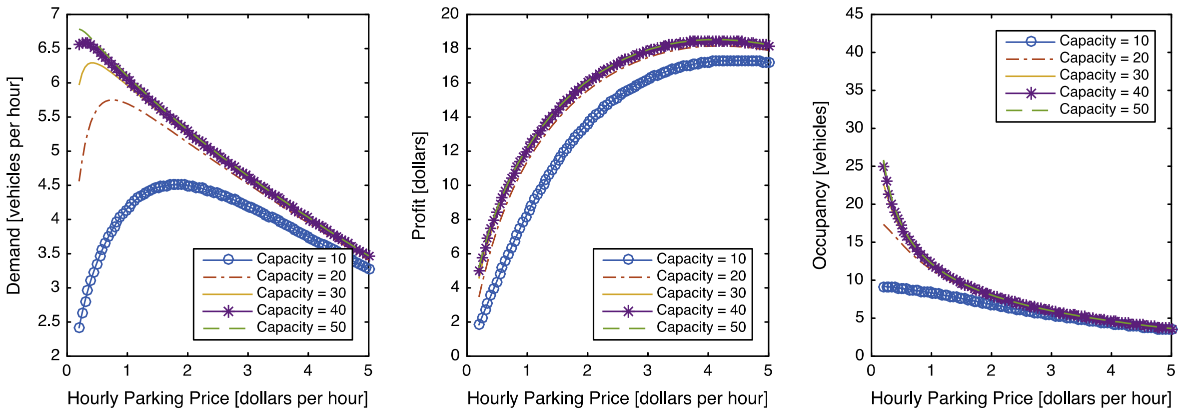

Fig. 4. Demand, profit, and occupancy for Scenario II with inelastic dwell time.

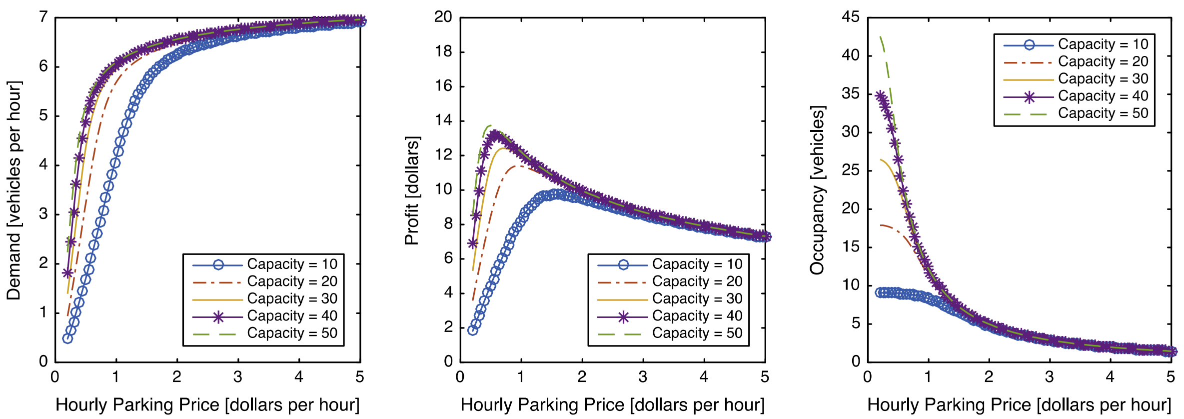

Fig. 5. Demand, profit, and occupancy for Scenario III with elastic dwell time.

The demand and profit for Scenarios I, II, and III are illustrated in Figs. 3–5, respectively. For each scenario, demand and profit are plotted for five parking capacities. Before discussing the scenarios, let us redefine C^i_{rs} by substituting Eq. (1) and Eq. (2) into Eq. (3):

$$C^i_{rs} = \alpha t_{ri} + \beta \frac{l_i \mu_i k_i}{k_i - \sum_{(r,s)} d^i_{rs} h^i_{rs}} + (g_i + p_i h^i_{rs}) + \gamma w_{is} + \gamma w_{si} + \alpha t_{ir} \quad \forall (r,s) \in V, \forall i \in \Omega(r,s) \tag{35}$$

As is now shown in Eq. (35), h_{rs}^i generally influences C_{rs}^i in two separate terms (second and third terms of Eq. (35)). However, under Scenario I, given that $p_i h_{rs}^i = p_i 3 p_i^{-1} = 3$ is a constant, h_{rs}^i influences C_{rs}^i only via the second term. Hence, as p_i increases, h_{rs}^i decreases causing C_{rs}^i and consequently d_{rs} to approach their asymptotic values as is shown in Fig. 3. The profit of this scenario also reaches its asymptotic value for the same reason. In Scenario II, demand initially increases with price and then decreases as is shown in Fig. 4. The initial increase occurs because increasing p_i leads to a lower dwell time and lower generalized cost, which in turn increases demand as elaborated in Section 3. The latter decrease in demand occurs because p_i directly contributes to the generalized cost which reduces demand. The demand in Scenario III somewhat follows the same pattern as Scenario I (as shown in Remark 2 of Section 3) but the profit patterns are different as shown in Fig. 5. In Scenario III, the profit reaches a peak value due to the higher influence of price on reducing dwell time. In all three scenarios, cases with higher parking capacities have higher demand, profit, and occupancy due to the lower cost of searching for parking (second term of Eq. (35)). Moreover, for all parking capacities in all three scenarios, demand, profit, and occupancy converge. The reason of convergence is that at high p_i values, dwell time and parking occupancy become so low that the parking capacity no longer imposes any restriction.

6.2. A network example

The second network is a grid network with 32 origins (external) nodes, 49 destination (internal) nodes, and 64 parking areas as shown in Fig. 6. The network includes a total of 144 bidirectional traffic links and a total of 196 bidirectional walk paths that connect the parking areas to the final destination zones. Travel time on each walking link is fixed and equal to 5 min but the travel time of each traffic link is obtained from the BPR function $t = f[1 + (x/cap)^4]$ where $f = 5$ min is the free-flow travel time and *cap* = 1000 vehicles per hour is the capacity of each link. The parking search time at each parking area is obtained from the BPR-type function $F(q) = \mu l[1 + (q/k)^2]$ where $\mu = 0.5$ min $l = 1$ and $k = 30$ vehicles is the capacity of each parking area. The dispersion parameter in the stochastic equilibrium model is set to $\theta = 0.9$. The demand function is $D_{rs}(\eta_{rs}, z) = 9(1 - z/20)\exp(-0.07\eta_{rs})$ where z is the distance from node r to the geometrical center of the network. The demand function leads to higher demand in the center of the of network, thus replicating a CBD. The travelers of all OD pairs are assumed to homogenous. Parking dwell times are obtained from $H_{rs}(p_i) = 1.5p^{e_p^h}$ where e_p^h is the dwell time elasticity to hourly parking price.

The parking search time and demand are depicted in Fig. 7 when dwell time is inelastic (i.e., $-1 < e_p^h \leqslant 0$). As illustrated, increasing the hourly parking price, p, from \$1 to \$3, reduces parking search time because dwell time is shorter at $p = \$3$ and more spots are available. This reduction in parking search time increases demand because of the lower generalized cost of

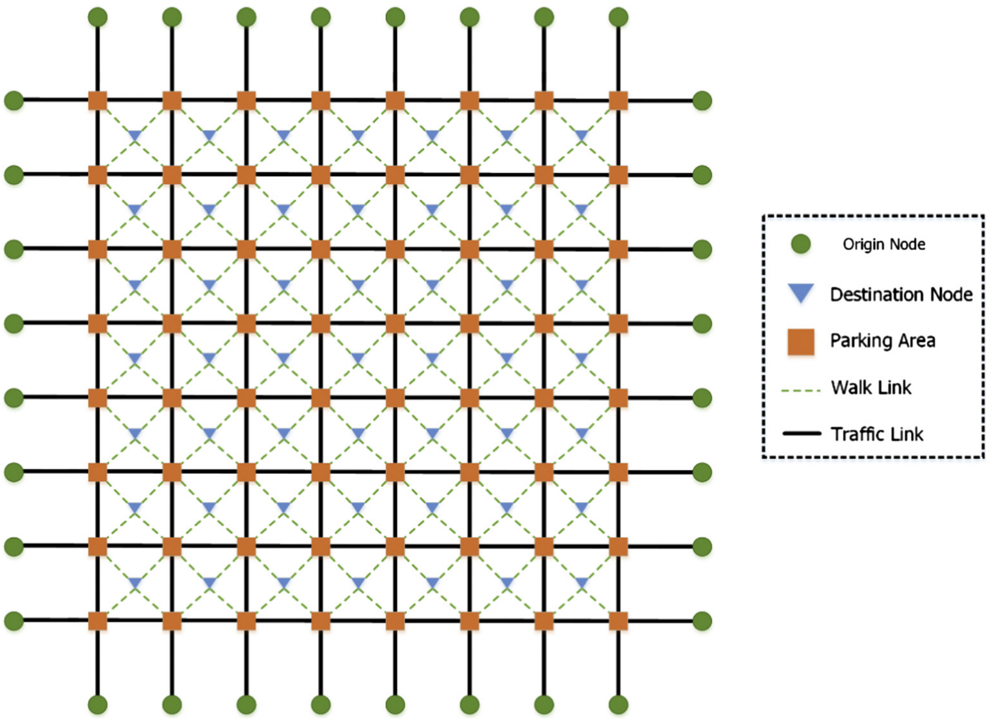

Fig. 6. Example network.

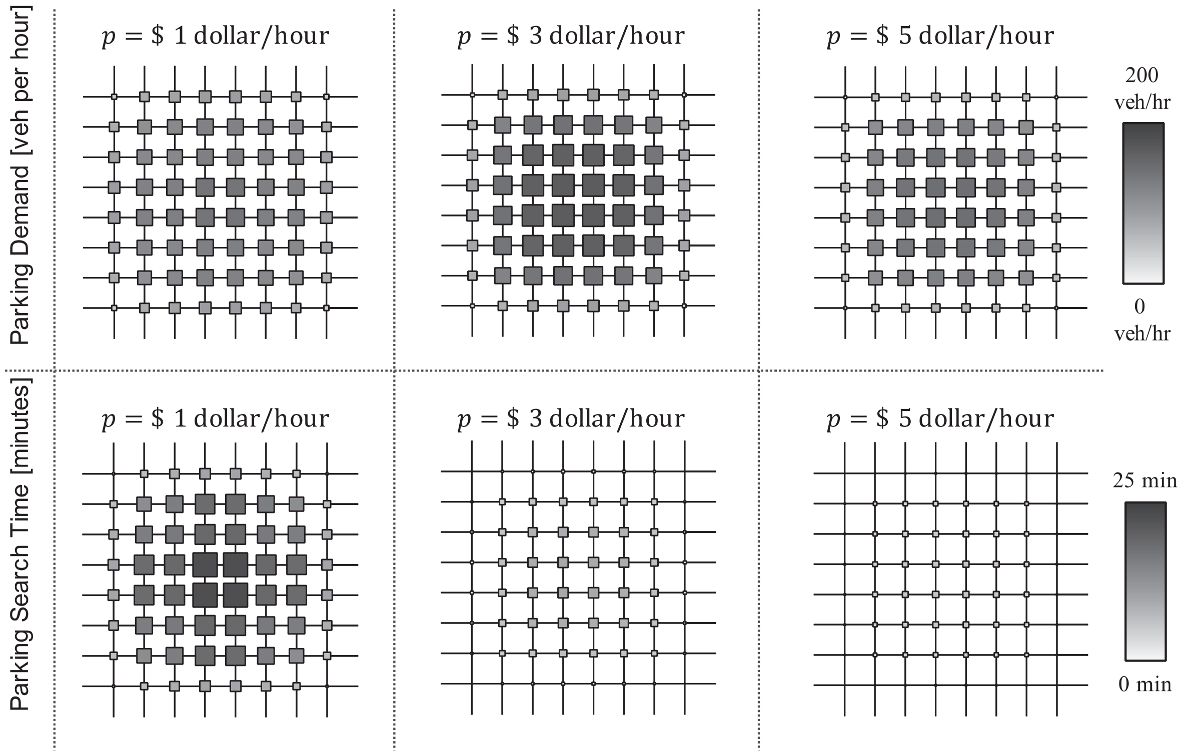

Fig. 7. Parking demand and search time for inelastic dwell time with $e_p^h = -0.3$.

travel. From $3 to $5, on the other hand, parking demand decreases because the low parking search time is offset by the high hourly price of $5 dollars per hour. To sum up, when dwell time is inelastic, the hourly parking price may increase or decrease demand but the parking search time always decreases. This remark is consistent with Proposition 1 which was proved for a simple case with only one origin, one destination, and one parking area. The parking search time and demand for the elastic case (i.e., $e_p^h < -1$) are presented in Fig. 8. As illustrated, increasing p strictly decreases search time and increases demand. This remark is also consistent with Proposition 1.

To investigate the impact of hourly parking price on parking behavior, Fig. 9 illustrates the average parking search time and demand (for the 64 parking areas) for a range of dwell time elasticities and hourly parking prices. As shown, when dwell time elasticity, e_p^h, is very low and close to zero, parking demand strictly decreases with p. This shows that at $e_p^h \approx 0$, the hourly parking price has the same impact as a road toll on demand as they both decay demand. However, when $-1 < e_p^h < 0$, parking demand first increases then decreases with p, and when $e_p^h < -1$, parking demand strictly increases with p. Fig. 9 also shows that search time always decreases when p regardless of the elasticity.

Fig. 10 shows the scatter of parking demand and search time for inelastic dwell time (Fig. 10a and b) and elastic dwell time (Fig. 10c and d). It is evident from Fig. 10b and d, that search time has a lower standard deviation when dwell time is elastic (compared to the inelastic case) because drivers are more sensitive to the hourly parking price. Precisely, the high sensitivity to p leads to a faster reaction (of drivers) to hourly parking prices. For the same reason, the mean of search time (as marked in the box plots) reaches zero at a faster rate when dwell time is elastic. Parking demand is shown to be fairly constant in the inelastic case shown in Fig. 10a (although it slightly increases and then decrease with p) but it increases at a fast rate in the elastic case. Finally, Fig. 11 shows the convergence of the algorithm by depicting demand at each parking area at 8 iterations of the algorithm for $e_p^h = -0.5$.

7. Conclusions

This paper investigates the impact of hourly parking pricing on travel demand. Parking pricing, if imposed wisely, has the potential to complement or even substitute road pricing. When imposed imprudently, however, it can increase demand and create more congestion. This paper shows that road pricing and hourly parking pricing are structurally different in how they influence the traffic equilibrium. While road pricing reduces demand, parking pricing can reduce or induce demand depending on the elasticity of parking dwell time to the hourly parking price. Hence, neglecting the dwell time elasticity can lead to suboptimal pricing and reduced social-welfare. To capture the impact of the hourly price on demand, a simple case is

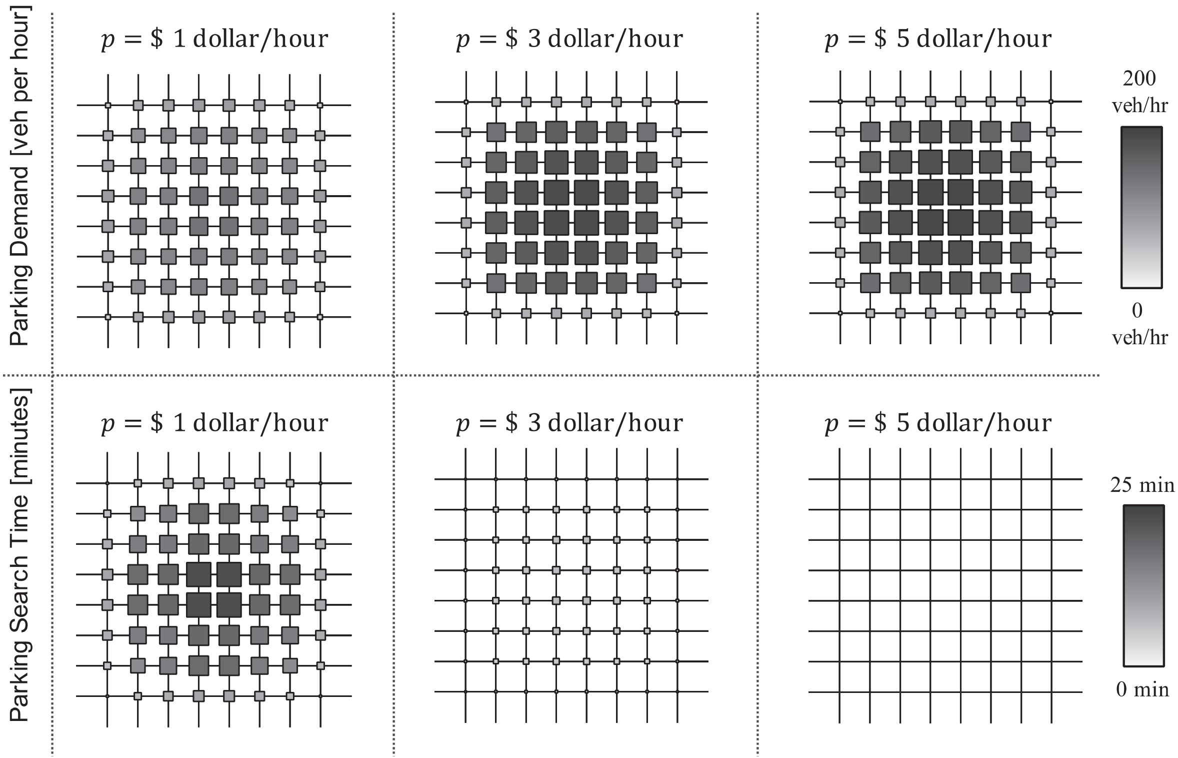

Fig. 8. Parking demand and search time for elastic dwell time with $e_p^h = -1.3$.

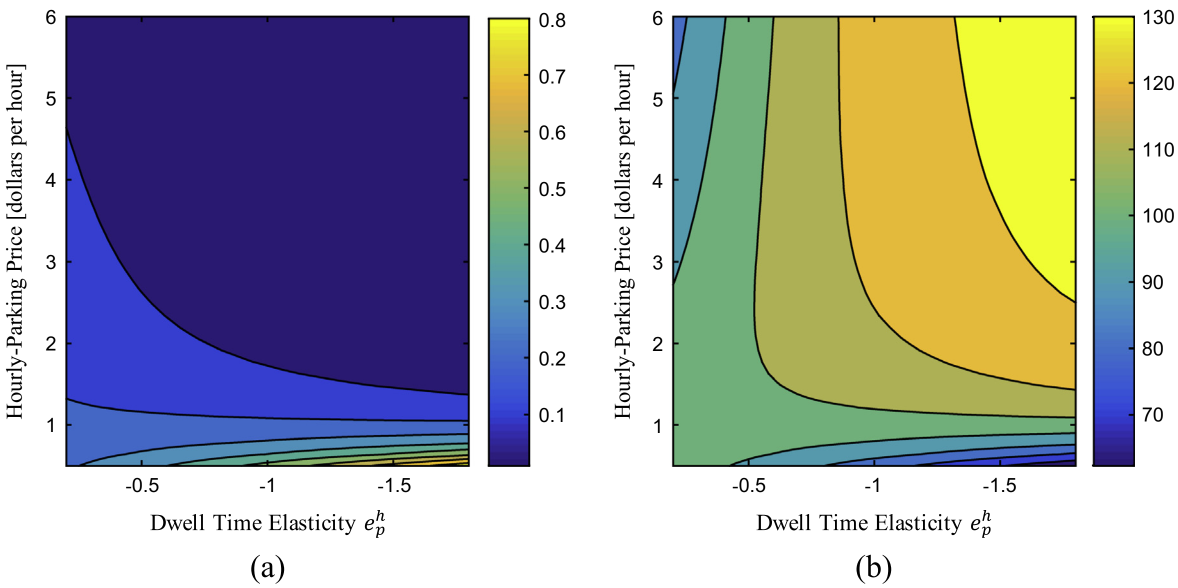

Fig. 9. (a) Average parking search time [hours], and (b) average parking demand [vehicles per hour].

presented with one origin, one destination, and one parking area. For this simple case, we prove that increasing the hourly parking price always induces demand when dwell time is highly elastic to the parking price. On the other hand, when dwell time is inelastic to parking price, an increase in the hourly parking price may increase or decrease demand. Hence, dwell time elasticity requires special attention in design of parking policy.

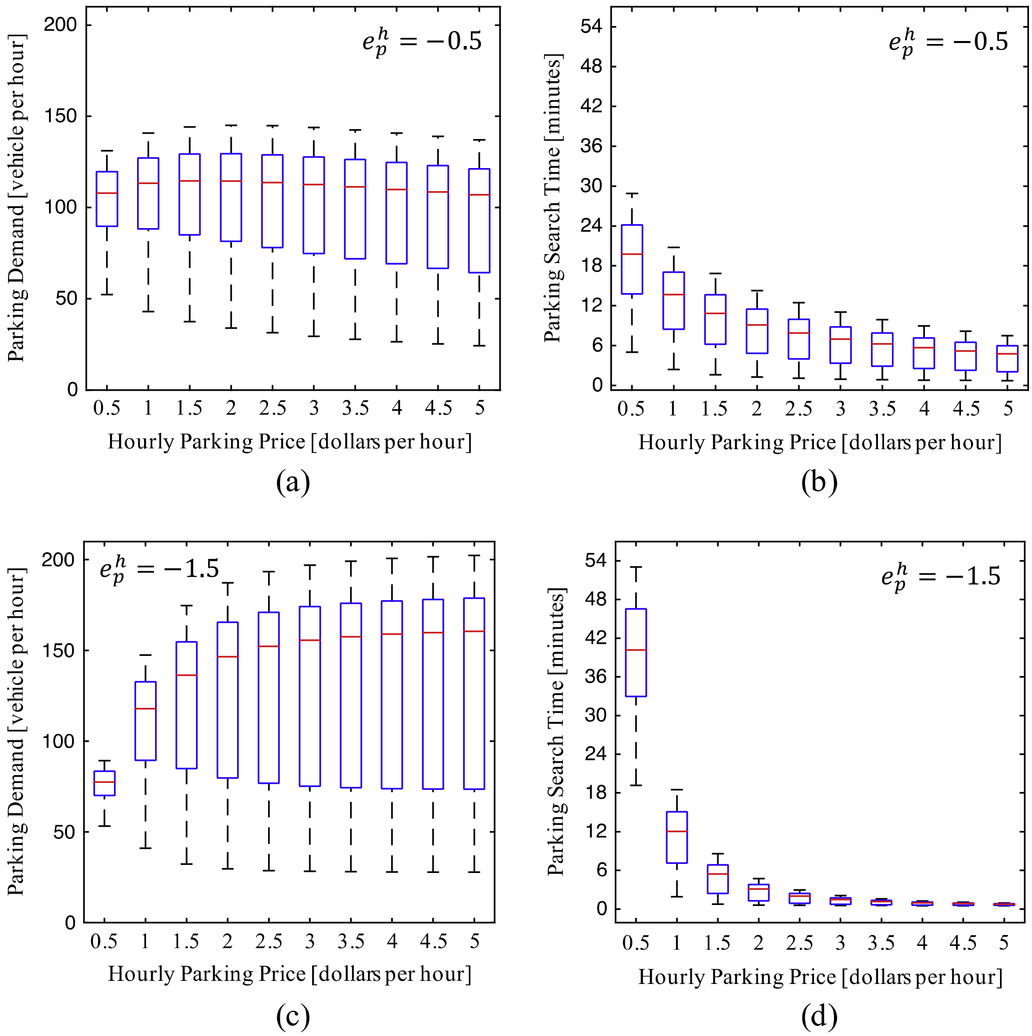

Fig. 10. (a) Parking demand, and (b) search time for inelastic dwell time ($e_p^h = -0.5$). (c) parking demand, and (d) search time for elastic dwell time ($e_p^h = -1.5$).

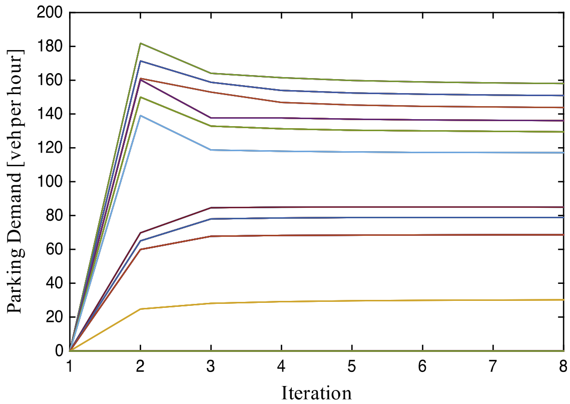

Fig. 11. Demand at each parking area at eight iterations of the algorithm.

For more realistic networks, we present a Variational Inequality model that captures the parking (and route) choice equilibrium. To gain managerial insight, we perform sensitivity analysis on a network with 64 parking areas. Numerical results show that the dwell time elasticity is still a key factor in travel demand. When dwell time elasticity is equal to zero, the hourly parking price has the same impact as a road toll on travel demand where increasing the parking price decreases demand. The numerical experiments also show a lower standard deviation in the parking search time (i.e., time to find a parking spot) of the 64 parking areas when dwell time is highly elastic to the hourly parking price. Moreover, when dwell time is inelastic, demand is fairly constant, whereas when dwell time is elastic, demand is asymptotic.

In conclusion, the main finding in this paper is that parking pricing policies should be devised with sufficient knowledge of dwell time elasticities. While this paper emphasizes the role of dwell time in parking policy, there are several aspects that are worthy of future research. First, user heterogeneity should be addressed by segmenting travelers based on their value of time and dwell-time elasticity. In a heterogeneous setting, demand of each traveler segment is sensitive to that segment's dwell time elasticity as well as the dwell time elasticity of all other segments. Second, with the growing interest in dynamic parking pricing in many major cities, the model can be modified with hourly prices changing within a day. This implies that parking occupancies are dynamic as well. Third, non-linear pricing can further improve social welfare where the hourly parking price increases (or decreases) with time. Finally, the impact of hourly parking pricing is amplified in the presence of multiple public and private parking management authorities who are generally in competition with each other (Arnott and Rowse, 2009, 2013). This leads to competitive price-setting environment which requires further research when prices are hourly-based.

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