Transportation Research Part A 102 (2017) 33–50

Contents lists available at ScienceDirect

Transportation Research Part A

journal homepage: www.elsevier.com/locate/tra

Parking enforcement policies for commercial vehicles

Mehdi Nourinejad *, Matthew J. Roorda

Department of Civil Engineering, University of Toronto, 35 St. George Street, Toronto, ON M5S 1A4, Canada

ARTICLE INFO

Article history:
Available online 21 May 2016

Keywords:
Parking
Enforcement
Freight
Inspection
Meeting function

ABSTRACT

Commercial vehicles are of particular interest in parking enforcement because of their heavy presence in central business districts and their recurrent behavior of illegal parking. To deter illegal commercial vehicle parking, enforcement policies are defined by the citation fine and level of enforcement. This paper investigates how rational carriers react to a policy under steady state equilibrium conditions. To model the equilibrium, the paper uses the theory of bilateral searching and meeting where enforcement units meet illegally parked commercial vehicles at a rate which depends on the size of the two agents (illegally parked commercial vehicles and enforcement units). In assessing policy effectiveness, two objectives are defined which are profit maximization and social cost minimization. With the two objectives, the paper presents three market regimes and studies the equilibrium of each market. The proposed model covers several gaps in the parking literature by introducing illegal parking behavior elasticity with respect to parking dwell time, level of enforcement, citation fine, and citation probability. The model is applied on a case study of the City of Toronto and the results show that the citation probability increases with dwell time and the level of enforcement. Increasing either the citation fine or level of enforcement will hinder illegal parking but the obtained profit remains approximately constant. Sensitivity analysis on the meeting rate elasticity shows that profits are low when both elasticities are either high or low.

1. Introduction

Illegal parking leads to adverse societal impacts such as reduced traffic speeds, loss of revenue from legal parking, and more accidents caused by safety violations. In response to these detrimental consequences, policies are imposed to alleviate illegal parking. Parking enforcement, the most prevalent policy, has been implemented in major cities for many years. Clearly, an effective enforcement plan requires an in-depth understanding of the causes and patterns of illegal parking. Commercial vehicles (CV) are of particular interest in parking enforcement because of their heavy presence in central business districts and their recurrent behavior of illegal parking. In 2014 alone, a total of 691,240 tickets were issued to CVs in Toronto, Canada, almost a quarter of the total number of parking tickets (City of Toronto, 2015). The CV tickets generated $30,516,000 for the city which the carriers are willing to pay as part of the high cost of the "last mile" in the supply chain. To exacerbate the situation, illegally parked CVs create other problems such as increased traffic delay and unsafe conditions. Estimates show that illegal CV parking results in approximately 47 million vehicle-hours of delay each year in the United States, making illegal CV parking the third leading cause of delay behind construction and crashes (Han et al., 2005).

* Corresponding author.
E-mail addresses: mehdi.nourinejad@mail.utoronto.ca (M. Nourinejad), roordam@ecf.utoronto.ca (M.J. Roorda).

http://dx.doi.org/10.1016/j.tra.2016.04.007

Moreover, CVs commonly park on bike lanes in order to reduce their egress time to the delivery destinations. In New York City, an average of 14% of CV on-street parking results in a conflict with a cyclist (Conway et al., 2013). With all these complications, parking enforcement policies must be designed with consideration of illegal CV parking.

The three fundamental components of any parking enforcement policy are detection technology, level of enforcement, and the citation fine. Detection technology is the method of finding illegally parked CVs and the two prominent methods are human[1] surveillance and video detection (Mithun et al., 2012); level of enforcement is the density of the enforcement units (e.g. cameras or on-foot officers) in the region; and citation fine is the imposed penalty for illegal parking. While the choice of the detection technology is a long term decision, policy-makers generally have more power over choosing the level of enforcement and the citation fine. The City of Toronto, for instance, has practiced human surveillance since the initiation of its enforcement policy but has changed the citation fine many times such as in 2015 when parking fines during peak periods were raised from $60 to $150 and the number of parking enforcement officers were increased as well (Powell and Clarke, 2015). Hereafter, we use the term "policy" to refer to a chosen level of enforcement and the citation fine.

Enforcement policies can influence the parking behavior of CVs. A large fine can deter CVs from parking illegally whereas a small fine may be considered by carriers as "the cost of doing business" (Nourinejad et al., 2014). Similarly, a high level of enforcement increases the probability of receiving a citation thus discouraging illegal CV parking. For the city, choosing the right policy depends on what objective is pursued. Two common objectives are profit maximization and social cost minimization. The citation profits are substantial in many cities. In 2013, New York City, Los Angeles, and Chicago each generated 534, 250, and 176 million dollars, respectively. In some instances, target profits are defined annually and policies are devised to reach them. Profit maximization must consider the reactive behavior of CVs as well. For instance, increasing the fine does not always lead to a higher profit because some CVs might start to park legally in order to avoid the penalty. As the second objective, social cost is seldom quantified but equally important. The extra traffic delay that CVs generate is cumbersome for society. The two objectives are not naturally obtained from one policy. The policy that maximizes profits may compromise social welfare. In this paper, we formulate the two objectives, model the reactive behavior of CVs, and present the tradeoff between the two objectives.

This paper is organized as follows. A review of research on illegal parking is presented in Section 2 and the gaps in the literature are highlighted. A model of parking enforcement with special treatment of CVs is presented in Section 3. In Section 4 two policy objectives are formulated and three market regimes are discussed. Numerical experiments are performed in Section 5 using the City of Toronto as a case study. Conclusions are presented in Section 6.

2. Background

Despite the abundance of research on parking enforcement, most studies provide only a descriptive analysis of illegal parking such as on-street parking meter behavior (Adiv and Wang, 1987), illegal parking behavior in central business districts (Bradley and Layzell, 1986; Brown, 1983), impact of illegal parking on local businesses (May, 1985), impact of parking fines on public transportation ridership (Auchincloss et al., 2014), CV illegal parking behavior (Wang and Gogineni, 2015; Wenneman et al., 2015), and non-CV illegal parking in loading bays (Aiura and Taniguchi, 2005; Alho and e Silva, 2014). A review of descriptive models is presented in one (if not the only) literature review of parking enforcement by Cullinane and Polak (1992) that focuses on the relationship between illegal parking and parking controls and the factors that influence the choice of illegal parking.

Descriptive models, although useful for identifying the factors that influence illegal parking behavior, do not provide a tool for finding the optimal enforcement policy. The need for prescriptive parking enforcement models is advocated in a recent literature review by Inci (2014) where the need for theoretical modeling techniques is stressed. In theory, the parking enforcement problem is an inspection game where the enforcement units are the inspectors and the CVs are the inspectees (Ferguson and Melolidakis, 1998; Avenhaus and Canty, 2012; Sasaki, 2014). In the classical inspection game, there are two players called the inspector and the inspectee. The inspector's strategy space is to audit the inspectee or not and the inspectee's strategy space is to violate the rules or not. The conditional probability that a violating inspectee is caught (i.e. the citation probability) is equal to the audit probability of the inspector. In the illegal parking problem, however, the citation probability (i.e. probability of catching a violating inspectee) is a function of how long the illegal CV parks (i.e. the dwell time) and the number of enforcement units. An increase in either the level of enforcement or the dwell time of the illegally parked CV increases the citation probability. This feature of the illegal parking problem merits an appropriate modeling approach.

To accommodate this feature in the inspection game, we use the concept of bilateral searching and matching (or bilateral meeting) which models the meeting friction between two sets of agents. Examples of bilateral meeting in economics include taxi-passenger meeting (Yang et al., 2010), buyer–seller meeting (Burdett et al., 2001), and employer–employee meeting in the labor market (Andolfatto, 1996; Berman, 1997). The meeting function is formulated so that the meeting rate of the two agents is a function of the number of agents. For instance, the rate of taxis meeting passengers depends on how many vacant taxis and passengers are available (Yang et al., 2010). A review of the bilateral meeting function is conducted by Petrongolo and Pissarides (2001). In this paper, we use the bilateral meeting function to model the searching friction present in the

[1] Human surveillance includes on-foot, cycling, and driving officers.

inspector–inspectee (i.e. enforcement unit – illegally parked CV) agents. The presented analytical formulation has two advantages. First, compared to other descriptive black box models, it clarifies the interplay between the factors that influence illegal parking behavior, citation probability, and the meeting rate. Second, it provides a quantitative tool for assessing optimality of policies.

Although the meeting function has never been used to model the parking enforcement problem as an inspection game, some studies have used other approaches to develop prescriptive solutions. Elliot and Wright (1982) study the relationship between parking compliance and enforcement and show that the relationship can be potentially unstable due to presence of hysteresis. In their proposed hysteresis theory, the inspection rate decreases with decreasing compliance simply because each enforcement unit spends more time citing the illegally parked CVs and has less time for inspection. Petiot (2004) extends the original parking model of Arnott and Rowse (1999) to model parking violations where each driver makes a binary choice of parking legally or illegally. If parked illegally, there is a fixed probability of receiving a citation. The binary choice is made in order to maximize the obtained utility of parking. The model captures the impact of the citation fine on illegal parking behavior but is not sensitive to parking duration or the level of enforcement. Kladeftiras and Antoniou (2013) present a microsimulation model to analyze the effects of illegal parking on traffic congestion and show that average traffic speeds can be increased by 10–15% if double parking is limited and that it can be increased by up to 44% if completely eliminated. The simulation model, although detailed in capturing parking dynamics, is developed only for modeling double-parking and does not capture the impact of enforcement on illegal parking behavior. Moving away from policy, Summerfield et al. (2015) develop a model for parking enforcement by formulating a Chinese Postman Problem to optimize the routing of the enforcement officers to maximize the total number of citations and minimize the total distance traveled by the officers. The model of Summerfield et al. (2015) is useful as a tool for operational decision-making but cannot be used for policy-making.

As it is evident from these studies, no research has yet been dedicated to modeling the relationship between parking dwell time, citation probability, and illegal parking behavior. Moreover, no research yet investigates the quantitative influence of parking enforcement policies (defined by a citation fine and the level of enforcement) on illegal parking behavior and the role of policies in helping cities reach their objectives of profit maximization and social cost minimization. Finally, there are few analytical models on parking enforcement that consider explicitly the parking patterns and delivery features of CVs (Alho, 2015; Alho et al., 2016). The model presented in Section 3 is developed to address these gaps in the literature.

3. Model

3.1. Problem setting

Notation	
$I = \{1, \ldots, i, \ldots, n\}$	set of n carriers
T_i	shipment frequency of carrier i [deliveries per hour]
F_i	fleet size of carrier i
C_i	capacity of each vehicle in the fleet of carrier i [shipping units]
X_i	vector of dwell time distribution parameters for carrier i
d_i	dwell time distribution for carrier i
$g_i(d_i; X_i)$	probability density function of the carrier i''s dwell time
w_i	average walking cost of carrier i if parked legally [Dollars]
f	fine of parking illegally [Dollars]
$\alpha(d)$	probability that an illegally parked vehicle is cited with dwell time d
p	cost of parking legally
G_i^l	expected cost of all legal parking for carrier i
G_i^v	expected cost of all illegal parking for carrier i
m	rate of parking enforcement units finding illegally parked vehicles
N^v	total number of illegally parked vehicles
N^e	total number of enforcement units
γ_1	elasticity of the meeting rate with respect to the enforcement units
γ_2	elasticity of the meeting rate with respect to the illegally parked vehicles

A total of n carriers defined by the set $I = \{1, \ldots, i, \ldots, n\}$ deliver daily shipments to a service area. The shipment frequency of each carrier $i \in I$ is denoted by T_i deliveries per hour. Each carrier $i \in I$ owns a fleet of F_i CVs, each with a capacity of C_i shipping units. Hence, for a given operating time (shift time) of S hours, we have:

$$F_i = ST_i/C_i \quad \forall i \in I \tag{1}$$

where ST_i is the total number of deliveries for carrier $i \in I$. Eq. (1), as it stands, assumes that CVs are filled to capacity. This assumption is not restrictive because C_i, which is physical capacity of the CVs, can easily be replaced with the effective capacity $\overline{C}_i$ (where $\overline{C}_i \leqslant C_i$) which is the actual used capacity of each CV. Since there is no logical interaction between parking enforcement policies and CV capacity, we assume $\overline{C}_i = C_i = C, \forall i \in I$, without any loss of generality.

Each delivery of each CV has a dwell time. The parking dwell time of carrier i's deliveries follows a continuous distribution $d_i \sim D(X_i)$ where X_i is the vector of distribution parameters and $g_i(d_i; X_i)$ is the pdf of the distribution. Each CV of each carrier $i \in I$, when close to its delivery destination, chooses to park legally or illegally. If parked legally, the CV pays a fixed parking fee of p dollars (to park as long as required to load or unload) and an average walking cost of w dollars (product of walking distance and the value of time).[2] The walking cost can also be carrier specific so that w_i is the average walking cost of carrier $i \in I$. If parked illegally, the CV does not pay a parking fee but will have to pay a penalty of f dollars if cited by parking enforcement. It is assumed that illegally parked CVs (hereafter referred to as illegal CVs) park so close to their destination that their walking distance is equal to 0. That is, all legal parking is far-sided and all illegal parking is near-sided. In cases where there is an average walking distance with illegal parking as well, then w_i can be interpreted as the difference in the walking cost of parking legally and illegally.

The probability that an illegal CV is cited is denoted by $\alpha(d)$ which is a function of the CV's parking duration d. The citation probability $\alpha(d)$ is strictly increasing; the longer the parking duration, the higher the probability of getting a citation. Hence, for a given dwell time d, the expected cost of parking illegally is $f\alpha(d)$ and the cost of parking legally is $p + w$. The two costs are illustrated in Fig. 1. As depicted, the cost of legal parking is fixed regardless of the parking duration[3] whereas the expected cost of illegal parking increases with dwell time and it converges to f when $d \to \infty$. This indicates that an illegal CV is bound to get a citation if parked for a very long time. Proposition 1 shows that p and f should be selected so that $p + w \leqslant f$, otherwise the legal parking supply will be underutilized.

Proposition 1. *The parking fee p and citation fine f should be chosen so that the inequality* $p + w \leqslant f$ *holds.*

Proof. As mentioned earlier, the expected cost of legal and illegal parking are $p + w$ and $f\alpha(d)$, respectively. Hence, the maximum cost of illegal parking is f when $\alpha(d) \to 1$. Now, assume the opposite of Proposition 1 is true so that $p + w > f$. In this case, every CV would park illegally because even under the toughest enforcement conditions with $\alpha(d) = 1$, the CV still pays less for illegal parking. The inequality $p + w > f$ indicates an underutilization of the legal parking supply which is not logically sound. Hence, $p + w \leqslant f$. □

The following Lemma is now imposed.

Lemma 1. *At every delivery, each carrier* $i \in I$ *makes a choice of parking legally or illegally based on the duration of that stop. When the dwell time* d_i *is shorter than some threshold* $\bar{d}_i$ *the carrier parks illegally. Alternatively, the carrier parks legally when the dwell time d is longer than* $\bar{d}_i$.

Proof. Under steady state equilibrium conditions, $\bar{d}_i$ is chosen so that $f\alpha(\bar{d}_i) = p + w$ as illustrated in Fig. 1. Given that the cost of illegal parking $f\alpha(d)$ is strictly increasing, for $d > \bar{d}_i$ we have $f\alpha(d) > p + w$. Hence, a CV with $d > \bar{d}_i$ will park legally due to its lower cost. However, when $d \leqslant \bar{d}_i$ we have $f\alpha(d) \leqslant p + w$. Hence, a CV with $d \leqslant \bar{d}_i$ will park illegally due to its lower cost. The rationale for Lemma 1 is that a CV is less likely to be cited for illegal parking when the parking duration is short.□

Let G_i^l and G_i^v denote the total cost of parking legally and illegally (the superscript 'l' represents legality and the superscript 'v' represents violation), respectively. According to Lemma 1, every CV of carrier $i \in I$ with dwell time $d > \bar{d}_i$ will park legally due to their lower cost. Hence, G_i^l is calculated as:

$$G_i^l = \int_{\bar{d}_i}^{\infty} (p + w) T_i g_i(v) dv \quad \forall i \in I \tag{2}$$

where the bounds of the integral represent the domain of parking duration for legal parking. In Eq. (2), the term $T_i g_i(v)$ is the parking frequency of carrier $i \in I$ with dwell time v, dv is the infinitesimal dwell time, and the term $p + w$ is the cost per delivery. Hereafter we set $p = 0$. This assumption does not compromise the generality of the equations because Eq. (2) is still dependent on w. In other words, eliminating p from Eq. (2) is equivalent to choosing a larger w that incorporates the cost p as well.

According to Lemma 1, every CV of carrier $i \in I$ with dwell time $d \leqslant \bar{d}_i$ will park illegally. Hence, G_i^v is calculated as:

$$G_i^v = \int_0^{\bar{d}_i} f\alpha(v) T_i g_i(v) dv \quad \forall i \in I \tag{3}$$

[2] The term w can also include the parking search time associated with legal parking.

[3] The assumption of a fixed price for legal parking does not compromise the generality of the model. This cost could also be a function of dwell time which makes the equations more complex without adding any insight.

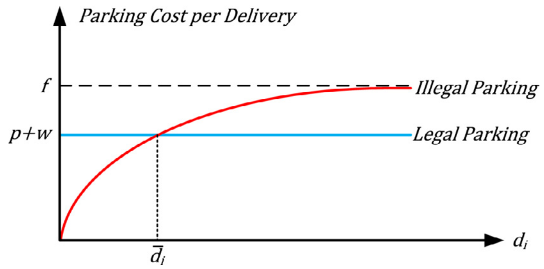

Fig. 1. Cost of legal and illegal parking with respect to dwell time.

where $f\alpha(v)$ is the cost of an illegal CV for duration of v and $T_i g_i(v)$ is the parking frequency of carrier i with dwell time v. By the law of total expectation, the expected cost of each carrier $i \in I$ is denoted by G_i which is the sum of legal and illegal parking costs:

$$G_i = G_i^l + G_i^v \quad \forall i \in I \tag{4}$$

The objective of each carrier $i \in I$ is to minimize G_i via the proper selection of $\bar{d}_i$. It is clear that the optimal $\bar{d}_i$ occurs at the point where $f\alpha(\bar{d}_i) = p + w$ as indicated by Lemma 1.

3.2. Citation probability function

The following assumptions are imposed. An inspection can lead to a citation if the CV is illegally parked and has not already been cited. If a CV is caught multiple times by enforcement units during its dwell time, it receives only one citation. Each vehicle is prone to receiving a citation for illegal parking regardless of how many citations it received on its former deliveries.

Let the term "meet" define the event where an enforcement unit inspects an illegally parked CV. The meeting rate is measured in CVs per hour and denoted by m. Given that the meetings are independent from each other, it can be assumed that the citation process follows a Poisson distribution and the inter-arrival time between each two meetings, denoted by the continuous variable t, follows an Exponential distribution. Using the law of random incidence (Larson and Odoni, 1981), the probability that an illegal CV with dwell time d is cited is equal to the probability that the citation inter-arrival time is smaller than or equal to d. Hence, the citation probability $\alpha(d, m)$ is calculated as:

$$\alpha(d, m) = Pr(t \leqslant d) = 1 - e^{(-md)} \tag{5}$$

Eq. (5) shows that the citation probability depends on both the dwell time and the meeting rate m. Note that Eq. (5) has the strictly increasing feature that was considered in Lemma 1. Eq. (5) also indicates a citation probability of 1 when the dwell time is very long, i.e. $\lim_{dto\infty}\alpha(d) = 1$, which is rationally correct since a CV with a very long dwell time is bound to get a citation.

The meeting rate m depends on the following three factors: (i) the level of enforcement, (ii) the number of illegal CVs, and (iii) the enforcement technology. The enforcement technology is assumed to be fixed and defined. Let N^e denote the number of enforcement units (where the superscript 'e' refers to enforcement) homogenously scattered in the service area and let N^v denote the total number of illegal CVs in the service area. The citation rate can formally be defined as:

$$m = M(N^e, N^v) \tag{6}$$

where the function M is defined so that $\partial M/\partial N^e > 0$ and $\partial M/\partial N^v > 0$ which indicates that increasing the enforcement N^e or illegal CVs N^v causes the meeting rate to rise. Furthermore, $\lim_{N^v \to 0} m = 0$ and $\lim_{N^e \to 0} m = 0$, indicating that no meetings will occur when there are no illegal CVs in the service area to be cited or when there is no enforcement available. Moreover, let

$$\gamma_1 = \frac{\partial m}{\partial N^e}\frac{N^e}{m} \tag{7}$$

$$\gamma_2 = \frac{\partial m}{\partial N^v}\frac{N^v}{m} \tag{8}$$

represent the elasticity of the meeting rate with respect to N^e and N^v, respectively. We have $\gamma_1, \gamma_2 > 0$ indicating that increasing either N^v or N^e leads to a higher meeting rate. The elasticities are later used in the sensitivity analysis of Section 5.

Calculation of N^v and N^e is elaborated as follows. The enforcement level N^e is a policy decision made by the enforcement authority (the city) and the number of illegal CVs N^v is dependent on the parking behavior of all the carriers. To obtain N^v, the number of each carrier's illegal CVs at the steady state equilibrium conditions must be calculated in the following way. There are $T_i g_i(v)$ deliveries per hour belonging to carrier i whose dwell times are between v and $v + dv$. The product of the delivery rate $T_i g_i(v)$ and the dwell time v (which is equal to $vT_i g_i(v)$) gives carrier i's number of deliveries at steady state equilibrium with dwell times that are between v and $v + dv$. Given a dwell time domain of $[0, \bar{d}_i]$ for illegal CVs of carrier i, the expected number of illegally parked carrier i CVs, denoted by N_i^v, is calculated as

$$N_i^v = \int_0^{\bar{d}_i} \frac{vT_i g_i(v)}{C_i} dv \quad \forall i \in I \tag{9}$$

Note that C_i (capacity of carrier i CVs) in the denominator of Eq. (9) converts the number of deliveries into the number of CVs. Since C_i and T_i are both constants, we drop for brevity from hereafter and reformulate Eq. (9) as[4]:

$$N_i^v = \int_0^{\bar{d}_i} vT_i g_i(v) dv \quad \forall i \in I \tag{10}$$

The total number of all illegal CVs, the summation of N_i^v across all carriers, is calculated as:

$$N^v = \sum_{i \in I} N_i^v = \sum_{i \in I} \int_0^{\bar{d}_i} vT_i g_i(v) dv \tag{11}$$

3.3. Equilibrium conditions

So far we have shown that each carrier i minimizes its cost G_i by choosing $\bar{d}_i$ at which the cost of legal parking w_i is equal to the expected cost of illegal parking $f\alpha(\bar{d}_i, m)$. Thus, for each carrier i, we have:

$$w_i = f\alpha(\bar{d}_i, m) \quad \forall i \in I \tag{12}$$

where $\alpha(\bar{d}_i, m)$ is obtained from Eq. (5). With a bit of simplification and using Eq. (5), Eq. (12) can be rewritten as:

$$\bar{d}_i = \frac{-\ln\left(1 - \frac{w_i}{f}\right)}{m} \quad \forall i \in I \tag{13}$$

which shows that $\bar{d}_i \forall i \in I$ can be obtained if the meeting rate m is known. For a fixed level of enforcement N^e, the meeting function itself only depends on N^v which according to Eq. (10) is a function of the vector of dwell time thresholds of all carriers. In mathematical form, with a fixed N^e, we have

$$m = M(\bar{d}_i, \forall i \in I) \tag{14}$$

The equilibrium occurs at the vector $\bar{d}_i \forall i \in I$ where Eqs. (13) and (14) are simultaneously satisfied. The existence conditions of the equilibrium are presented in Proposition 2.

Proposition 2. *There exists an equilibrium solution if the meeting rate m is always bounded from below and above such that* $\check{m} \leqslant m \leqslant \hat{m}$.

Proof. Let $\bar{\boldsymbol{d}} = (\bar{d}_i, \forall i \in I)$ be the vector of carrier dwell time thresholds and let Ω represent the feasible set of $\bar{\boldsymbol{d}}$. Brouwer's fixed-point theory states the following: If $\Gamma : \Omega \to \Omega$ is a continuous function mapping a compact and convex set Ω into itself, then there is some vector $\bar{\boldsymbol{d}}$ in Ω such that $\bar{\boldsymbol{d}} = \Gamma(\bar{\boldsymbol{d}})$. Hence, in order to prove the existence of a solution, we need to show that Γ is compact and convex.

To show that Γ is compact, we have to show that it is closed and bounded. According to Eq. (13) and given the condition $\check{m} \leqslant m \leqslant \hat{m}$, it is easy to infer that the following condition holds:

$$\frac{-\ln\left(1 - \frac{w_i}{f}\right)}{\hat{m}} \leqslant \bar{d}_i \leqslant \frac{-\ln\left(1 - \frac{w_i}{f}\right)}{\check{m}} \tag{15}$$

where the upper-bound of Eq. (15) occurs at $m = \check{m}$ and the lower-bound occurs at $m = \hat{m}$. Given Eq. (15), it is clear that Γ is bounded and closed and hence compact.We now show that Γ is convex by showing that its second derivative is positive. With some simplification, the second derivative of Eq. (13) is positive when the following condition is justified

$$m \leqslant 2 \frac{dm}{d\bar{d}_i} \bigg/ \frac{d^2 m}{d\bar{d}_i^2} \quad \forall i \in I \tag{16}$$

[4] Dropping the capacity C_i from Eq. (9) is equivalent to defining T_i as the number of deliveries per vehicle per hour.

By choosing the upper-bound of m such that $\widehat{m} = \min_i\left\{2\frac{dm}{d\bar{d}_i}\Big/\frac{d^2m}{d\bar{d}_i^2}\right\}$, the second derivative of Eq. (13) will always be positive and Γ will always be convex. □

In finding the equilibrium of the highly non-linear system, we use Brouwer's fixed-point theory which is explained in Algorithm I (Fuente, 2000).

Algorithm I. Brouwer's fixed-point algorithm

Step 0 – Initialization: Set the iteration counter k to 0. Set the dwell time threshold of each carrier to randomly chosen value $\tilde{d}_i, \forall i \in I$, so that $\bar{d}_i^k = \tilde{d}_i, \forall i \in I$.

Step 1 – Update the meeting rate: given $\bar{d}_i^k$, use Eq. (14) to find the meeting rate m.

Step 2 – Update the iteration counter: Set $k \leftarrow k+1$.

Step 3 – Update the dwell time thresholds: given m from Step 1, use Eq. (13) to obtain the new vector of dwell time thresholds $\bar{d}_i^k, \forall i \in I$.

Step 4 – Convergence criterion: terminate the algorithm if the convergence criterion is satisfied. Otherwise go to Step 1. The convergence criterion is the following condition

$$\sum_{i\in I}(\bar{d}_i^k - \bar{d}_i^{k-1})^2 \leqslant \varepsilon \tag{17}$$

where ε is a predetermined convergence tolerance.

To ensure convergence, we use the following modified algorithm with the addition of the method of successive averages (MSA) to solve for the equilibrium. The modified algorithm uses MSA in Step 3 of Algorithm I. The revised Step 3 is presented below in Algorithm II. All other steps of Algorithm II are similar to those of Algorithm I. Algorithm II converges in all the empirical numerical experimentations.

Algorithm II. Method of successive averages algorithm

Step 3 – Update the dwell time thresholds: given m from Step 1, use Eq. (13) to obtain the new vector of dwell time thresholds $\bar{d}_i^{new}, \forall i \in I$. Use the method of successive averages to find the dwell time thresholds of the current iteration as

$$\bar{d}_i^k = \frac{k-1}{k}\bar{d}_i^{k-1} + \frac{1}{k}\bar{d}_i^{new} \tag{18}$$

3.4. Comparative static effects of regulatory variables

This section presents the comparative static effects of the regulatory variables N^e and f on transitional variables $\bar{d}_i$, m, and N^v and the intrarelationship between the transitional variables. The results of the static effects are presented in Eqs. (19)–(30) and their proofs are given in Appendix A. To avoid unnecessary detail, we have omitted the proofs that are too obvious.

The static effect of the regulatory variables on the transitional variables is presented through Eqs. (19)–(24).

$$\frac{dm}{dN^e} > 0 \tag{19}$$

$$\frac{dN^v}{dN^e} < 0 \tag{20}$$

$$\frac{d\bar{d}_i}{dN^e} < 0 \tag{21}$$

$$\frac{dN^v}{df} < 0 \tag{22}$$

$$\frac{dm}{df} < 0 \tag{23}$$

$$\frac{d\bar{d}_i}{df} < 0 \tag{24}$$

Eq. (19) shows that increasing enforcement raises the meeting rate m as more enforcement units would be searching for illegal CVs (Lemma 6). The increase in the meeting rate as a consequence of Eq. (19) will deter CVs from illegal parking which in turn leads to a lower N^v as is shown in Eq. (20) (Lemma 5). The decrease in N^v as a consequence of Eq. (20) is equivalent to carriers reducing their dwell time threshold $\bar{d}_i$ as shown in Eq. (21). Eq. (22) shows that increasing the citation fine lowers N^v as CVs would avoid illegal parking due to the higher penalty of getting a citation (Lemma 8). As N^v decreases, the meeting rate is lowered as well because there are less illegal CVs to be found as indicated in Eq. (23) (Lemma 7). Finally, the decrease in N^v as a consequence of Eq. (22) is equivalent to carriers reducing their dwell time threshold $\bar{d}_i$ as shown in Eq. (24).

The static intrarelationship between the transitional variables is presented through Eqs. (25)–(30).

$$\frac{d\bar{d}_i}{dm} < 0 \tag{25}$$

$$\frac{dN^v}{dm} < 0 \tag{26}$$

$$\frac{\partial m}{\partial N^v} > 0 \tag{27}$$

$$\frac{d\bar{d}_i}{dN^v} < 0 \tag{28}$$

$$\frac{\partial N^v}{\partial \bar{d}_i} > 0 \tag{29}$$

$$\frac{\partial m}{\partial \bar{d}_i} > 0 \tag{30}$$

The impact of the meeting rate m on N^v and $\bar{d}_i$ is shown in Eqs. (25) and (26). Increasing the meeting rate is equivalent to a higher citation probability (see Eq. (5)) which reduces $\bar{d}_i$ (Lemma 3) and consequently N^v (Lemma 4) as CVs would be less inclined to park illegally. The impact of N^v on $\bar{d}_i$ and m is shown in Eqs. (27) and (28). Increasing N^v raises the pool of illegal CVs leading to a higher rate m as shown in Eq. (27). As a consequence of Eq. (27) a higher meeting rate reduces $\bar{d}_i$ (according to Eq. (25)) thus justifying Eq. (28). The impact of $\bar{d}_i$ on m and N^e is shown in Eqs. (29) and (30). Eq. (29) is obvious (Lemma 2). As a consequence of Eq. (29), when N^v increases, the meeting rate increases as well due to more illegal CVs as shown in Eq. (30).

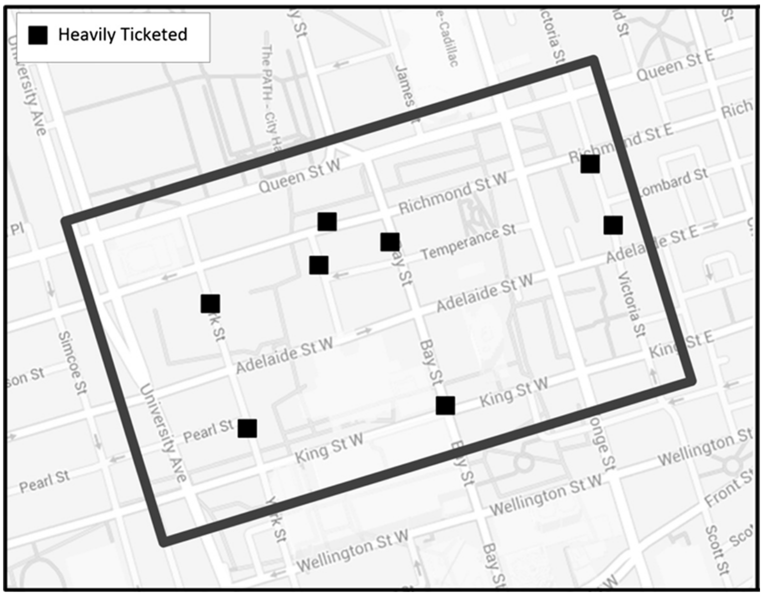

Fig. 2. Toronto's central business district.

Table 1
Company types.

Company Type	Delivery percentage	Description
Courier	30	Includes both corporate and smaller companies that offer package or mail deliveries
Food	24	Any catering company or wholesale food supplier or company specializing in utensils and tools related to food
Private	8	No visible label on the vehicle or dashboard of the associated company
Shredding	8	On-site shredding and data processing companies
Office Products	6	Office supplies such as paper, toner cartridges necessary for daily operations
Rental	4	Any vehicle labeled with a freight rental company name
Secure Courier	3	High security couriers employing armored trucks
Building Material	2	Deliver or provide construction materials such as steel
Cartage	2	Operators of large transport trucks specializing in LTL or TL deliveries
Cleaner	2	Laundromat companies that deliver clean or pick up dirty office clothes
Electronics	2	Any company that deliver electronic products
Furniture	2	Office furniture such as chairs and desks
Other	2	Other company types
Plants	2	Decorative plants and flowers
Sanitation	1	Sanitation products such as hand cleanser and toilet paper
Waste and Recycling	1	Waste collection and disposal or recycling companies
Mover	1	Office moving or relocating companies

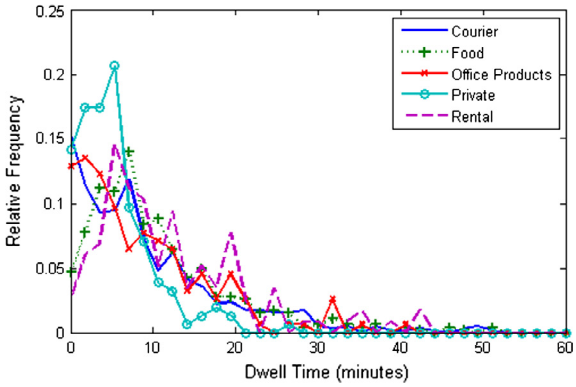

Fig. 3. Relative dwell time frequency of the main company types.

Table 2
Mean dwell time of company types.

Company type	Dwell time mean (minutes)	Dwell time variance
Courier	9.86	84.33
Food	15.91	99.21
Office Products	12.75	129.12
Private	9.34	51.45
Rental	9.65	39.05

4. Profit, social cost, and markets

The two objective functions of the enforcement authority are (i) profit maximization denoted by *PR* and (ii) social cost minimization denoted by *SC*. Profit is equal to the generated revenue from collecting citation fines minus the cost of enforcement (i.e. price of inspection units). Parking enforcement revenue itself is composed of the fines paid by each of the carriers that were cited. Let R_i denote the expected hourly revenue obtained from carrier $i \in I$ which is equal to

$$R_i = \int_0^{\bar{d}_i} f\alpha(v)T_i g_i(v)dv \quad \forall i \in I \tag{31}$$

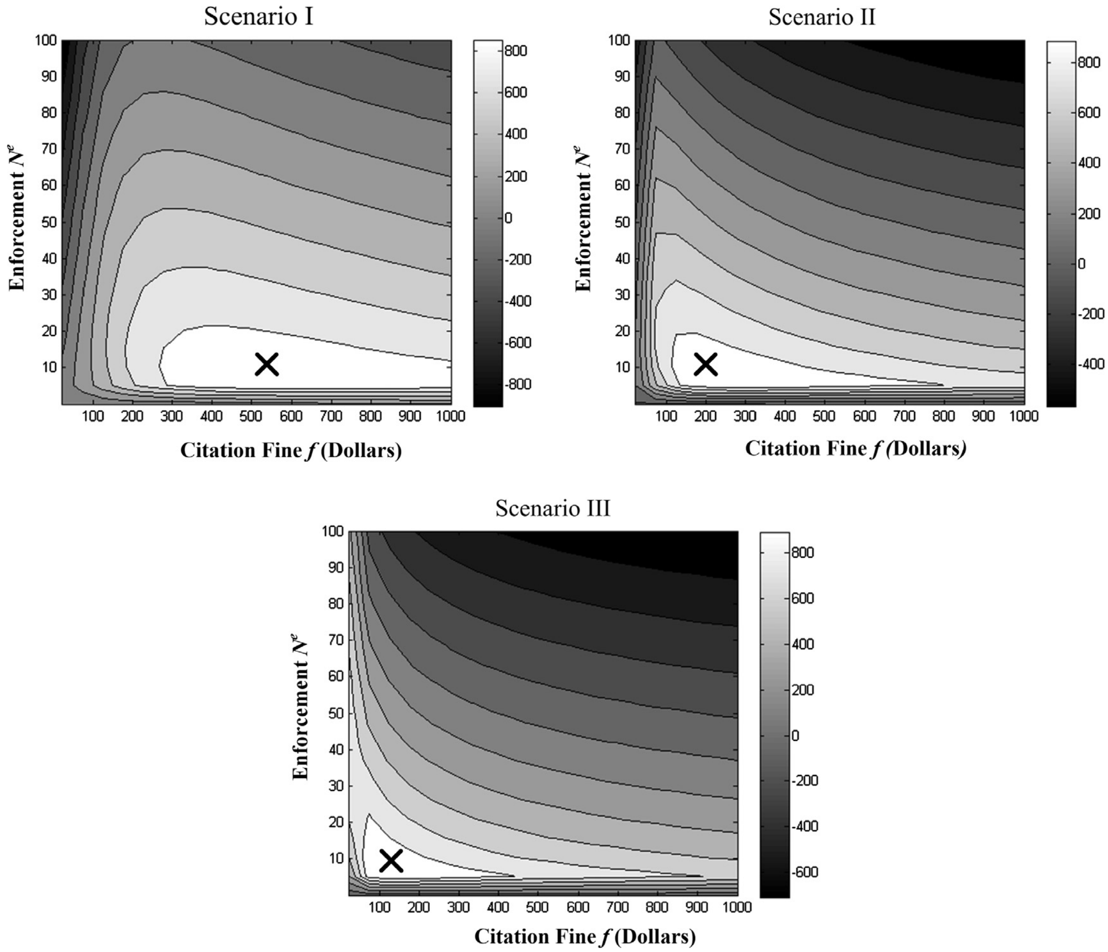

Fig. 4. Profit (Dollars) as a function of the citation fine and enforcement at three scenarios.

The total generated revenue, denoted by R, is equal to the sum of the revenue obtained from all carriers. Hence, revenue is computed as $R = \sum_{i \in I} R_i$ which is measured in dollars per hour. By letting β denote the cost of each enforcement unit per hour, the total cost of enforcement per hour is βN^e. The total profit from parking enforcement is the difference between the revenue and cost of enforcement which is given by

$$PR = R - \beta N^e \tag{32}$$

The social cost of each enforcement policy is composed of three negative externalities. The first component is the cost of enforcement which is equal to βN^e. The second component is the walking cost of the carriers associated with each legal parking delivery. The third component is the cost imposed by each illegal CV on through traffic. Let us assume that the negative externality of the extra travel time cost from each illegal CV on through traffic is δ dollars per illegal CV. Then social cost is computed as:

$$SC = \beta N^e + \sum_{i \in I} \int_{\hat{d}_i}^{\infty} w_i T_i g_i(v) dv + \delta N^v \tag{33}$$

where the second term on the RHS is the cost of walking for all carriers.

Assume that the city grants a single firm the monopoly rights of managing parking enforcement through policy-making that involves choosing a citation fine and the level of enforcement. Under this monopoly system, the single expected profit-maximizing firm would choose a policy that maximizes the generated profit. Hence, we have

$$\textit{Monopoly}: \ \text{Maximize } PR$$

Conventionally, the first-best social optimum policy is one at which the marginal social welfare benefit from adding one additional enforcement unit is equal to the marginal cost per enforcement unit. Given that the demand for parking is assumed to be fixed in this paper, we define the first-best social optimum as the policy at which social cost is minimized.

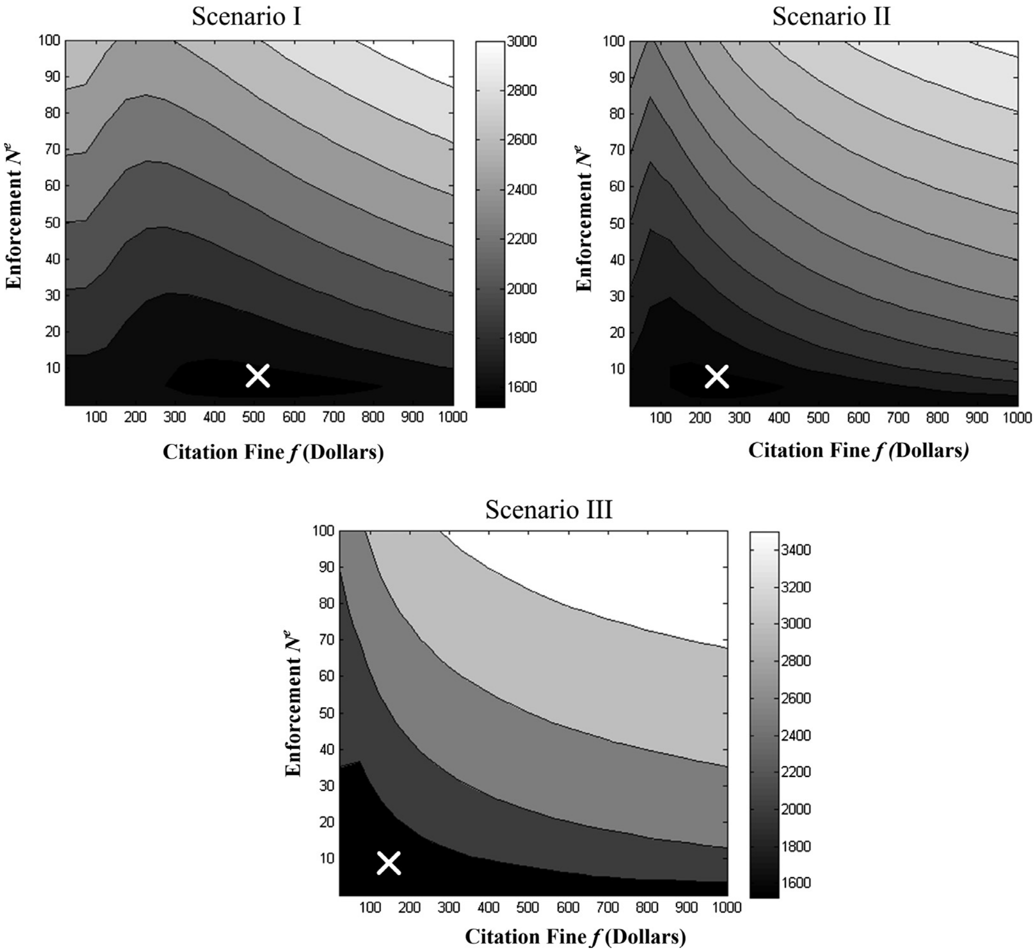

Fig. 5. Social cost (Dollars) as a function of the citation fine and enforcement at three scenarios.

$$\textit{First-best}: \ \text{Minimize } SC$$

The first-best social optimum policy may lead to a negative profit. Hence, the second-best social optimum regime is set up with the addition of the constraint that the generated revenue must cover the cost the enforcement (i.e. profits are nil). Despite the relevance of other second-best policies with $PR > 0$, we focus only of cases where $PR = 0$. The second-best market is defined as

$$\textit{Second-best}: \ \text{Minimize } SC$$

$$\text{Subject to}: \ PR = 0$$

5. Commercial vehicle parking in the City of Toronto

The central business district of the City of Toronto is chosen as a case study. The central business district, as shown in Fig. 2, is approximately 1 km by 1 km containing the highest employment density of Toronto with 8 of the 60 most heavily ticketed locations in Toronto reported by the Canadian Courier Logistics Association (Nourinejad et al., 2014). These locations are depicted by black squares in Fig. 2. A survey was conducted in the region in August 2010 to obtain information about carrier deliveries in Toronto's central business district (Kwok, 2010). The surveyor recorded details of carrier deliveries on individual road segments on weekdays between the hours of 9:00 AM and 3:00 PM. The surveyor collected the arrival time, departure time, parking location, type of location, and the company that owned the CV. A total of 1940 observations were recorded. Table 1 presents the percentage of the 17 company types that performed the deliveries. Among the 17 company types, the courier, food, private, office products, rental, and shredding companies have the largest presence. We focus only on first five prominent company types and ignore shredding companies mainly because shredding companies do not have a choice of parking legally as they are obliged by law to be on site (close to their destination) when shredding documents.

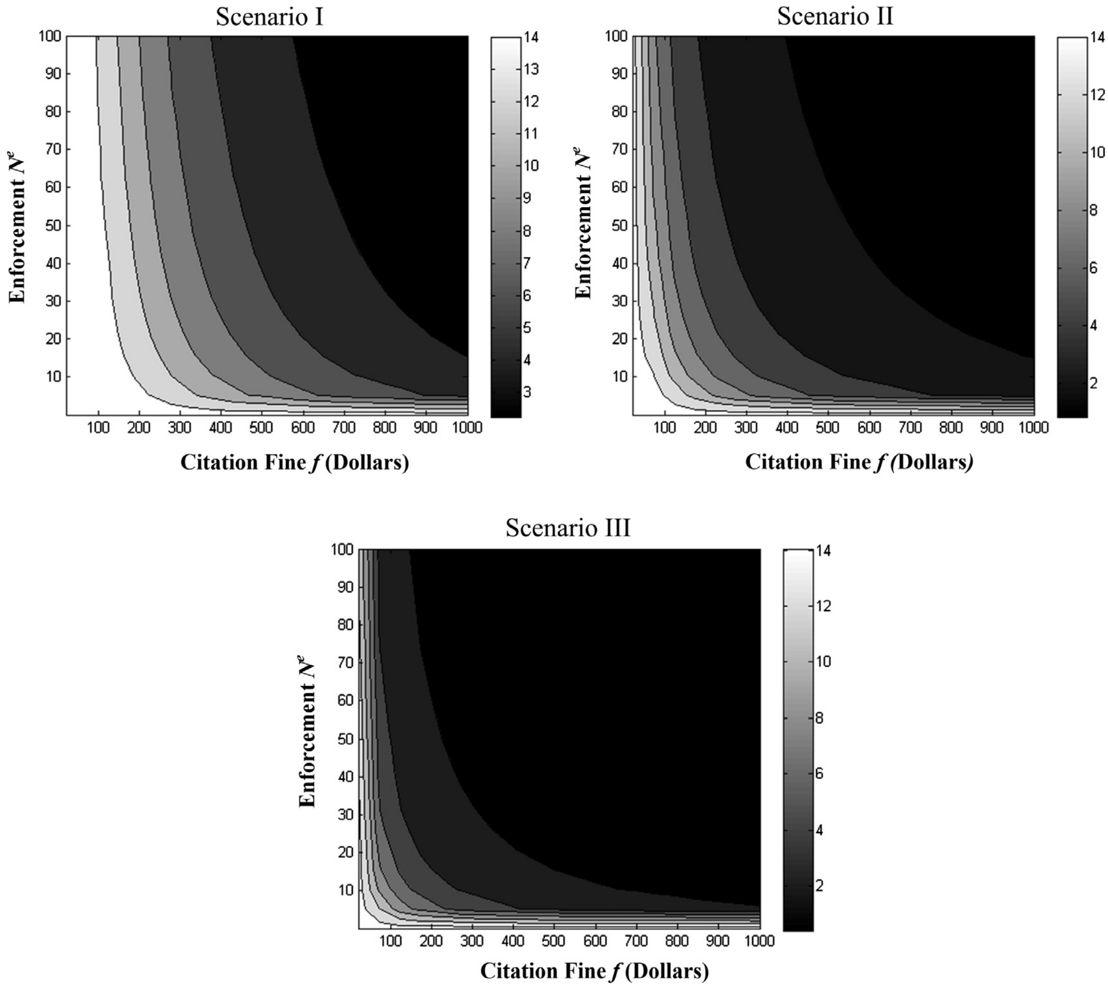

Fig. 6. Number of illegal CVs as a function of the citation fine and enforcement at three scenarios.

The relative dwell time frequency of the five company types is illustrated in Fig. 3 and the mean and variance of the dwell times are presented in Table 2. The dwell times are assumed to follow an exponential distribution[5]. All of the following analysis is performed in Matlab R2014.

The following costs are chosen for the case study. The cost of inspection β is set to 15 dollars[6] per enforcement unit per hour and the marginal social cost per each illegal CV (i.e. δ) is set to 250 dollars per illegal CV per hour. To obtain the walking cost w, we assume an average walking distance of 200 m, a walking speed of 5 km per hour, and a cost of 30 dollars per each hour of delay for each carrier.

Let us assume that meeting rate is obtained from a Cobb–Douglas type meeting function as is customary in the bilateral meeting literature (Varian, 1992, Yang et al., 2010; Yang and Yang, 2011). This function has the following form

$$M(N^v, N^e) = A(N^e)^{\gamma_1}(N^v)^{\gamma_2} \tag{34}$$

where A is a positive function parameter that depends on the spatial characteristics of the market and it can be negatively related to the size of the searching and meeting areas. As already mentioned, γ_1 and γ_2 are the elasticities of the meeting function with respect to N^e and N^v, respectively, and we have $0 < \gamma_1, \gamma_2 \leqslant 1$.

Consider the following three scenarios. In Scenarios I, II, and II, we set $\gamma_1, \gamma_2 = 0.3$, $\gamma_1, \gamma_2 = 0.5$, and $\gamma_1, \gamma_2 = 0.7$, respectively. Increasing γ_1 and γ_2 (when moving from Scenario I to Scenario III) is indicative of improved inspection technology. For each of the three scenarios, we have computed the city's profit (PR), social cost (SC), the meeting rate (m), and the number of illegal CVs (N^v). Fig. 4 illustrates the contours of the city's profit for a number of policies at each of the three scenarios.

[5] A Chi-squared goodness-of-fit test is conducted with a 90% confidence interval. All dwell times fit the exponential distribution without violating the null hypothesis of the test.

[6] All monetary units are in Canadian Dollars.

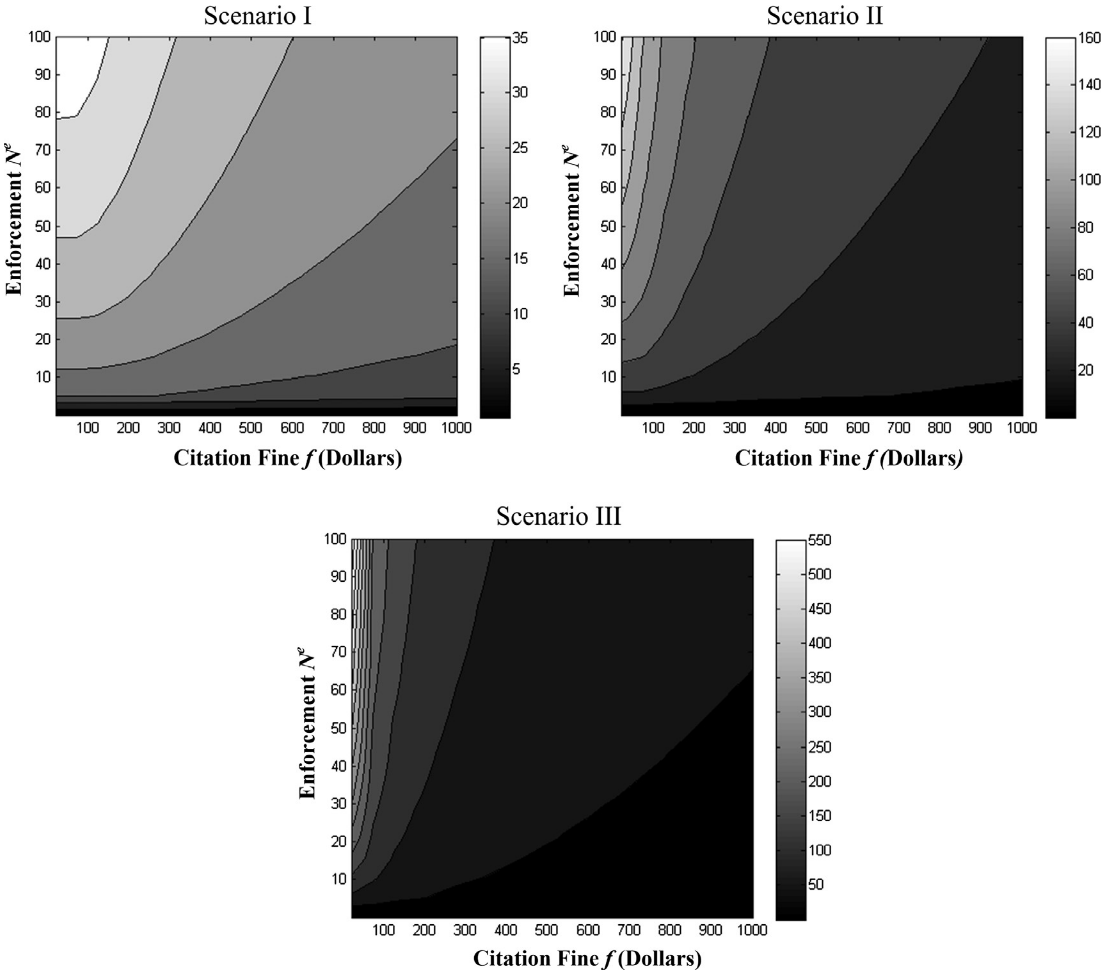

Fig. 7. Meeting rate (vehicles per hour) as a function of the citation fine and enforcement at three scenarios.

Each policy is defined by a given citation fine f and level of enforcement N^e. The horizontal axis on each panel of Fig. 4 represents the citation fine measured in dollars and the vertical axis represents the level of enforcement. The optimal policy for maximizing *PR* is shown by an ''X'' (which occurs at the peak of the contours) in each of the three scenarios. Fig. 4 shows that moving from Scenario I to Scenario III reduces the optimal citation fine f and enforcement N^e as a result of improved inspection and the higher meeting rate elasticities. However, the maximum profit remains approximately constant at 800 dollars per hour indicating that despite the improvement in the inspection technology, the reactive behavior of the CVs at each scenario defuses the potential of increasing the profit.

The social cost contours associated with each scenario are illustrated in Fig. 5. The optimal policy for minimizing *SC* is shown by an ''X'' sign (which occurs at the peak of the contours) in each of the three scenarios. In Scenario I, the city has to increase the citation fine substantially up to 500 dollars to compensate for the inefficient inspection as a result of a low γ_1 and γ_2. The increased citation fine discourages CVs from parking illegally without having to increase enforcement too much. Scenarios II and III, however, require a lower citation fine of 250 and 160 dollars per hour, respectively, due to improved inspection technology.

The number of illegal CVs at each scenario is illustrated in Fig. 6 where it is shown that an increase in the citation fine or enforcement lowers N^v as a result of the increased expected cost of parking illegally. Fig. 6 also shows that Scenario III, compared to the other two scenarios, has a lower N^v under all policies as a result of more efficient inspection. The meeting rate of the policies under each scenario is illustrated in Fig. 7 where it is shown that Scenario III has the highest meeting rate due to the larger meeting function elasticities. It is also evident from Fig. 7 that the meeting rate is positively related to enforcement and negatively related to the citation fine, thus validating the results of the comparative static effects of Section 3.4.

We now perform sensitivity analysis on the meeting function elasticities γ_1 and γ_2. In light of this, we assume a fixed and given citation fine of \$250 with a total of 20 enforcement units. The γ_1 and γ_2 values are both increased from 0 to 1. Profit, social cost, the meeting rate, and the number of illegal CVs are computed for each pair (γ_1, γ_2) and the results are illustrated in Fig. 8.

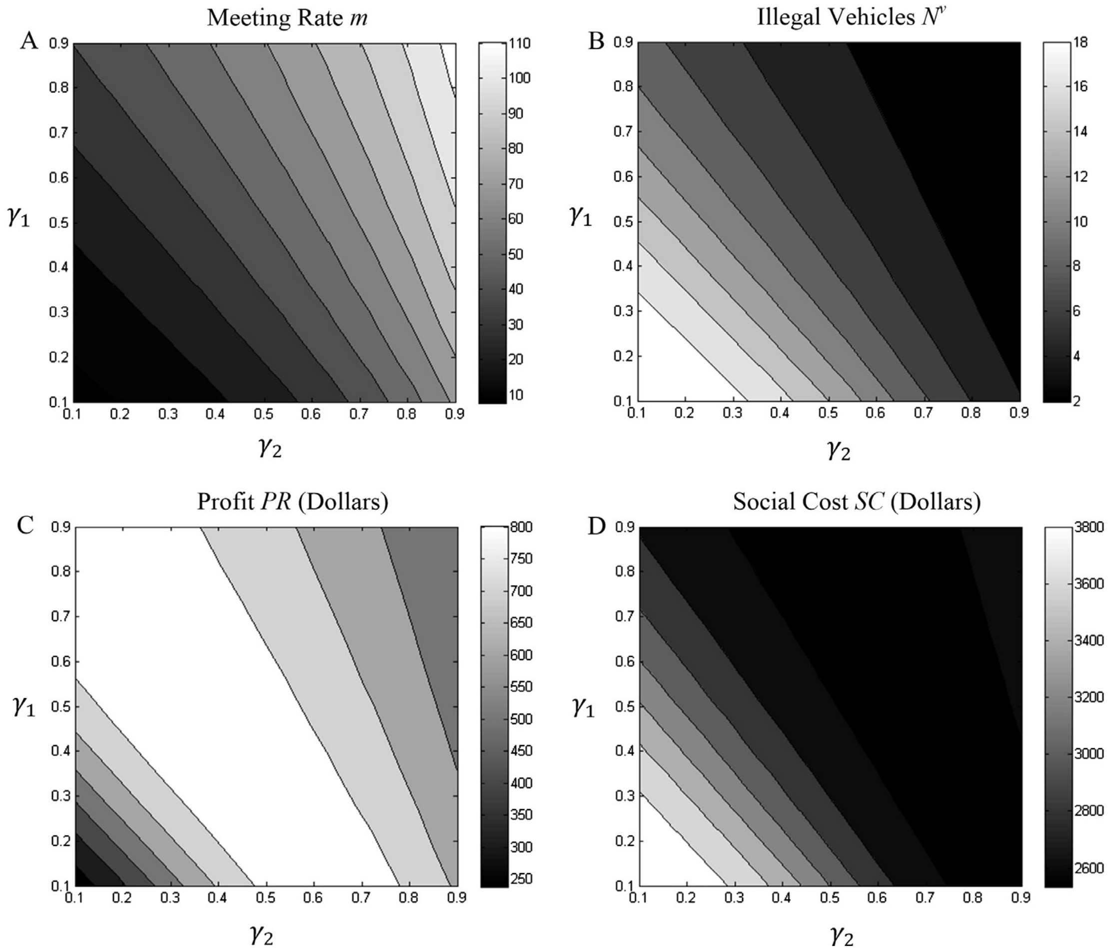

Fig. 8. Profit (Dollars), social cost (Dollars), illegal commercial vehicles (vehicles), and the meeting rate (vehicles per hour) as functions of meeting elasticity.

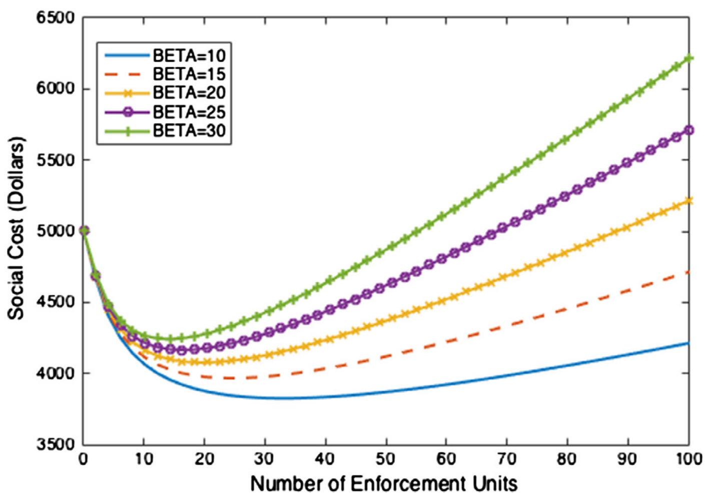

Fig. 9. Impact of the enforcement cost on social cost.

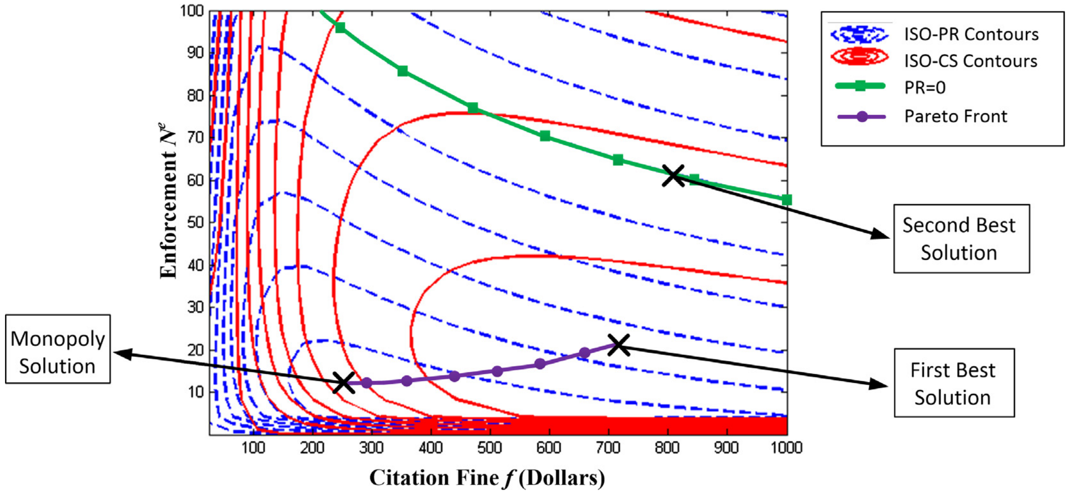

Fig. 10. Three parking enforcement markets.

We partition the elasticity space into three collectively exhaustive and mutually exclusive sets where γ_1 and γ_2 are both small, both large, and neither small nor large. Let us first analyze the part of the elasticity space where the γ_1 and γ_2 are both small. In this region, the meeting rate is small (Fig. 8A), due to the low values of γ_1 and γ_2, even though there are a lot of illegal CVs (Fig. 8B). Given the city's inability to cite the illegal CVs due to the small meeting rate, very little revenue is generated from illegal parking and profit becomes small (Fig. 8C). For the same reason, the high N^v leads to a large social cost due the induced congestion on through traffic (Fig. 8D). Let us now analyze the part of the elasticity space where the γ_1 and γ_2 are both large. When γ_1 and γ_2 are both large, the meeting rate is high due to its high sensitivity to N^e and N^v (Fig. 8A). The high meeting rate discourages illegal parking which leads to a low N^v (Fig. 8B) and a low profit as there are not enough illegal parked CVs to be cited (Fig. 8C). Analysis of the final segment of the elasticity space where γ_1 and γ_2 are neither low nor high shows that the highest profit is generated in this region since there are enough illegal CVs to be cited and the inspection technology is efficient enough to find and cite them.

Sensitivity analysis on the cost of an enforcement unit is presented in Fig. 9. Social cost is shown to first decrease and then increase with the number of enforcement units. The initial decease occurs because the enforcement units deter CVs to park illegally, thus lowering the delay cost on through traffic and consequently the social cost. The latter increase occurs because the cost of acquiring enforcement units no longer justifies the benefits of having fewer illegally parked CVs. Fig. 9 also shows that the optimal number of enforcement units (which occurs at the minimum social cost) decreases with β, thus indicating that more enforcement units should be acquired when they are inexpensive.

We now analyze the three defined markets of Section 4. Fig. 10 illustrates the iso-profit and iso-social cost contours for a number of policies with the assumption that $\gamma_1 = \gamma_2 = 0.4$. The equilibrium location of each market is identified in Fig. 10. The monopoly solution occurs at the policy with the maximum *PR* and the first best solution occurs at the policy with the minimum SC. Fig. 10 also identifies all policies at which the profit is nil (i.e. $PR = 0$). As mentioned earlier, the second best solution is a point on the $PR = 0$ line at which the social cost is minimal (Fig. 10). The second-best solution for this example is dominated by the first-best solution which leads simultaneously to a higher profit and social welfare. In addition to the three markets, the Pareto front of the two objective functions is illustrated in Fig. 10 as well.

6. Conclusions

This paper investigates how rational carriers react to parking enforcement policies under steady state equilibrium conditions. In modeling the equilibrium, this paper uses the concept of bilateral searching and meeting to capture the inherent friction in how parking enforcement units find illegally parked commercial vehicles. The use of the meeting function is helpful in capturing CV illegal parking sensitivity with respect to parking dwell time, level of enforcement, citation fine, and citation probability. The paper also introduces two objective functions for policy-making along with three parking enforcement market regimes. Sensitivity analysis is performed on a case study of the City of Toronto and the three market regimes are analyzed as well. Results show that the citation probability increases with dwell time and the level of enforcement. Increasing either of the citation fine or enforcement will hinder illegal parking but the obtained profit remains approximately constant. Sensitivity analysis on the meeting rate elasticity shows that profits are low when both elasticities are either high or low.

The City of Toronto has made major changes in its parking enforcement policy in 2015 by changing its previous revenue-based enforcement regime to one that is targeted at minimizing congestion especially during rush hours (CBC, 2015). Under the new policy, the citation fine has increased from $60 to $150 and intensive towing is implemented at the cost of the drivers – commercial vehicles, for instance, are charged $1000 for retrieval. This new enforcement blitz makes it substantially more difficult for courier and other commercial vehicles, that are under strict time windows, to complete their deliveries. The higher cost of delivery is in many cases considered the "cost of doing business" which eventually turns into higher user service costs and potentially lower social welfare. The prevalent tradeoffs that are present in any parking enforcement policy necessitate the need for models that quantify the impacts of parking enforcement policies. This paper presented a first attempt of quantifying the influential factors in parking enforcement to find the optimal policy.

While numerous insights are extracted from the presented model, there is still a need for addressing other sources of uncertainty (such as travel time, number of deliveries per day, and the choice of parking illegally), investigating the role of other policies, and validating the proposed models using empirical data. Addressing the sources of uncertainty, coupled with a rigorous survey of influential factors in the choice of parking, can lead to better calibrated models which are essential for policy making. In light of this, study of other parking enforcement policies such as towing, issuing parking permits, or wheel-clamping, provides authorizes with more flexibility when choosing the optimal policy that is best suited for the city.

Appendix A

The following lemmas are prepared as proofs of the relationships presented in Section 3.4.

Lemma 2. *For all* $\bar{d}_i$, *we have* $\frac{\partial N^v}{\partial \bar{d}_i} > 0$.

Proof. By taking the derivative of Eq. (10), we have $\frac{\partial N^v}{\partial \bar{d}_i} = \bar{d}_i T_i g_i(\bar{d}_i)$ which is a positive value since $\bar{d}_i, T_i, g_i(\bar{d}_i) > 0$. □

Lemma 3. *For all* m, *we have* $\frac{\partial \bar{d}_i}{\partial m} < 0$.

Proof. $\frac{\partial \bar{d}_i}{\partial m} = \ln\left(1 - \frac{w_i}{f}\right) \Big/ m^2$ (derivative of Eq. (13)) is a negative value. □

Lemma 4. *For all* m, *we have* $\frac{dN^v}{m} < 0$.

Proof. $\frac{dN^v}{m}$ can be rewritten as

$$\frac{\partial N^v}{\partial m} = \sum_i \frac{\partial N^v}{\partial \bar{d}_i} \cdot \frac{\partial \bar{d}_i}{\partial m} \tag{35}$$

where $\frac{\partial N^v}{\partial \bar{d}_i} = \bar{d}_i T_i g_i(\bar{d}_i)$ is a positive value according to Lemma 2 and $\frac{\partial \bar{d}_i}{\partial m} = \ln\left(1 - \frac{w_i}{f}\right) \Big/ m^2$ is a negative value according to Lemma 3, thus making $\frac{\partial N^v}{\partial m}$ negative. □

Lemma 5. *For all* N^e, *we have* $\frac{dN^v}{dN^e} < 0$.

Proof. Let $\frac{dN^v}{dN^e}$ be presented as:

$$\frac{dN^v}{dN^e} = \frac{dN^v}{dm} \frac{dm}{dN^e} \tag{36}$$

where $\frac{dm}{dN^e}$ (last term on the RHS of Eq. (36)) is

$$\frac{dm}{dN^e} = \frac{\partial m}{\partial N^e} + \frac{\partial m}{\partial N^v} \cdot \frac{dN^v}{dN^e} \tag{37}$$

Substituting Eq. (36) into Eq. (37) gives:

$$\frac{dN^v}{dN^e} = \frac{\partial N^v}{\partial m} \frac{\partial m}{\partial N^e} \Big/ \left(1 - \frac{\partial N^v}{\partial m} \frac{\partial m}{\partial N^v}\right) \tag{38}$$

Note that $\frac{\partial m}{\partial N^e} = \frac{m\gamma_1}{N^e}$ and $\frac{\partial m}{\partial N^v} = \frac{m\gamma_2}{N^v}$ are both positive in Eq. (38). Hence by showing that $\frac{\partial N^v}{\partial m} < 0$, which is proved in Lemma 4, it is clear that $\frac{dN^v}{dN^e} < 0$ for all N^e. □

Lemma 6. *For all N^e, we have $\frac{dm}{dN^e} > 0$.*

Proof. With some simplifications, $\frac{dN^v}{dN^e}$ can be calculated as:

$$\frac{dN^v}{dN^e} = \frac{-k\gamma_1}{\left(1 + \frac{k\gamma_2}{N^v}\right)N^e} \tag{39}$$

where $k = \sum_i T_i g_i(\bar{d}_i)\bar{d}_i^2 > 0$. Eq. (39) is consistent with Lemma 5 since it is strictly negative.

We now show through Lemma 6 the impact of enforcement on the meeting rate m.

By substituting Eq. (39) into Eq. (37), we have:

$$\frac{dm}{dN^e} = \frac{m\gamma_1}{\left(1 + \frac{k\gamma_2}{N^v}\right)N^e} \tag{40}$$

Given that $k > 0$, it is obvious from Eq. (40) that $\frac{dm}{dN^e} > 0$. □

Lemma 7. *For all f, we have $\frac{dm}{df} < 0$.*

Proof. Let $\frac{dm}{df}$ be presented as

$$\frac{dm}{df} = \frac{\partial m}{\partial N^v}\frac{dN^v}{df} \tag{41}$$

where $\frac{dN^v}{df}$ (the second term on the RHS of Eq. (41)) may be written as

$$\frac{dN^v}{df} = \sum_i \frac{dN^v}{d\bar{d}_i}\frac{d\bar{d}_i}{df} \tag{42}$$

Substituting Eq. (41) into Eq. (42) gives

$$\frac{dm}{df} = \frac{\partial m}{\partial N^v}\sum_i \frac{dN^v}{d\bar{d}_i}\frac{d\bar{d}_i}{df} \tag{43}$$

where $\frac{d\bar{d}_i}{df}$ (the second term on the RHS of Eq. (43)) is

$$\frac{d\bar{d}_i}{df} = -\left[\frac{dm}{df}\bar{d}_i + \frac{\alpha(\bar{d}_i)}{f(1-\alpha(\bar{d}_i))}\right] \Big/ m \tag{44}$$

Substituting Eq. (44) into Eq. (43) gives:

$$\frac{dm}{df} = \frac{-\frac{\partial m}{\partial N^v}\sum_i\left[\frac{dN^v}{d\bar{d}_i}\frac{\alpha(\bar{d}_i)}{f\left(1-\alpha\left(\bar{d}_i\right)\right)}\right]}{m + \frac{\partial m}{\partial N^v}\sum_i\left[\frac{dN^v}{d\bar{d}_i}\bar{d}_i\right]} \tag{45}$$

which is a negative number for all f. □

Lemma 8. *For all f, we have $\frac{dN^v}{df} < 0$.*

Proof. Let $\frac{dN^v}{df}$ be presented as

$$\frac{dN^v}{df} = \frac{dN^v}{dm}\frac{dm}{df} \tag{46}$$

According to Lemma 4, we have $\frac{dN^v}{m} < 0$ and according to Lemma 7 we have $\frac{dm}{df} < 0$. Hence, it is clear that $\frac{dN^v}{df} < 0$. □

References

Adiv, A., Wang, W., 1987. On-street parking meter behaviour. Transp. Quart., 281–307

Aiura, N., Taniguchi, E., 2005. Planning on-street loading-unloading spaces considering the behaviour of pickup-delivery vehicles. J. Eastern Asia Soc. Transp. Stud. 6, 2963–2974.

Alho, A., 2015. Improved Mobility and More Sustainable Urban Logistics Through the Configuration and Enforcement of (Un)loading Bays PhD Thesis. Universidade de Lisboa – Instituto Superior Técnico..

Alho, A.R., e Silva, J.D.A., 2014. Analyzing the relation between land-use/urban freight operations and the need for dedicated infrastructure/enforcement—application to the city of Lisbon. Res. Transp. Bus. Manage. 11, 85–97.

Alho, A.R., e Silva, J.D.A., de Sousa, J.P., Blanco, E., 2016. Decreasing congestion by optimizing the number, location and usage of loading/unloading bays for urban freight. In: Proceedings of the TRB 95th Annual Meeting.

Andolfatto, D., 1996. Business cycles and labor-market search. Am. Econ. Rev., 112–132

Arnott, R., Rowse, J., 1999. Modeling parking. J. Urban Econ. 45 (1), 97–124.

Auchincloss, A.H., Weinberger, R., Aytur, S., Namba, A., Ricchezza, A., 2014. Public parking fees and fines: a survey of US cities. Public Works Manage. Policy.

Avenhaus, R., Canty, M.J., 2012. Inspection Games. Springer, New York, pp. 1605–1618.

Berman, E., 1997. Help wanted, job needed: Estimates of a matching function from employment service data. J. Labor Econ., S251–S292

Bradley, M.A., Layzell, A.D., 1986. Parking Behaviour in a Suburban Town Centre. TSU, Oxford University.

Brown, M.B., 1983. Influences on Illegal Parking in Town Centres. Unpublished M.Sc. Thesis. Cranfield Institute of Technology.

Burdett, K., Shi, S., Wright, R., 2001. Pricing and matching with frictions. J. Polit. Econ. 109 (5), 1060–1085.

Canadian Broadcasting Corporation CBC, 2015. Zero Tolerance Rush Hour Policy Introduced to Downtown Drivers (accessed January, 2016).

City of Toronto, 2015. www.toronto.ca (accessed September, 2015).

Conway, A.J., Thuillier, O., Dornhelm, E., Lownes, N.E., 2013. Commercial vehicle-bicycle conflicts: a growing urban challenge. In: Transportation Research Board 92nd Annual Meeting (No. 13-4299).

Cullinane, K., Polak, J., 1992. Illegal parking and the enforcement of parking regulations: causes, effects and interactions. Transp. Rev. 12 (1), 49–75.

Elliot, J.R., Wright, C.C., 1982. The collapse of parking enforcement in large towns: some causes and solutions. Traffic Eng. Control 23 (6), 304–310.

Ferguson, T.S., Melolidakis, C., 1998. On the inspection game. Naval Res. Logist. 45 (3), 327–334.

Fuente, A., 2000. Mathematical Methods and Models for Economists. Cambridge University Press, Cambridge.

Han, L., Chin, S.M., Franzese, O., Hwang, H., 2005. Estimating the impact of pickup-and delivery-related illegal parking activities on traffic. Transp. Res. Rec.: J. Transp. Res. Board 1906, 49–55.

Inci, E., 2014. A review of the economics of parking. Econ. Transp.

Kladeftiras, M., Antoniou, C., 2013. Simulation-based assessment of double-parking impacts on traffic and environmental conditions. Transp. Res. Rec.: J. Transp. Res. Board 2390, 121–130.

Kwok, J., 2010. Data Collection on Parking and Loading Supply and Truck Driver Demand Survey. Final Report. September.

Larson, R.C., Odoni, A.R., 1981. Urban Operations Research.

May, A.D., 1985. Parking Enforcement: Are We Making the Best use of Resources. UTSG Conference Paper.

Mithun, N.C., Rashid, N.U., Rahman, S.M., 2012. Detection and classification of vehicles from video using multiple time-spatial images. IEEE Trans. Intell. Transp. Syst. 13 (3), 1215–1225.

Nourinejad, M., Wenneman, A., Habib, K.N., Roorda, M.J., 2014. Truck parking in urban areas: application of choice modelling within traffic microsimulation. Transp. Res. Part A: Policy Pract. 64, 54–64.

Petiot, R., 2004. Parking enforcement and travel demand management. Transp. Policy 11 (4), 399–411.

Petrongolo, B., Pissarides, C.A., 2001. Looking into the black box: a survey of the matching function. J. Econ. Lit., 390–431

Powell, B., Clarke, 2015. Toronto Mayor John Tory Proclaims Illegal Parking Blitz a Success. Toronto Star, January 2015.

Sasaki, Y., 2014. Optimal choices of fare collection systems for public transportations: barrier versus barrier-free. Transp. Res. Part B: Methodol. 60, 107–114.

Summerfield, N.S., Dror, M., Cohen, M.A., 2015. City streets parking enforcement inspection decisions: the Chinese postman's perspective. Eur. J. Oper. Res. 242 (1), 149–160.

Wang, Q., & Gogineni, S., 2015. An empirical investigation of commercial vehicle parking violations in New York City. In: Transportation Research Board 94th Annual Meeting (No. 15-5958).

Wenneman, A., Habib, K.M.N., Roorda, M.J., 2015. A disaggregate analysis of the relationships between commercial vehicle parking 1 citations, parking supply, and parking demand: critical understandings and policy 2 implications 3. In: Transportation Research Board 94th Annual Meeting (No. 15-1195).

Yang, H., Yang, T., 2011. Equilibrium properties of taxi markets with search frictions. Transp. Res. Part B: Methodol. 45 (4), 696–713.

Yang, H., Leung, C.W., Wong, S.C., Bell, M.G., 2010. Equilibria of bilateral taxi–customer searching and meeting on networks. Transp. Res. Part B: Methodol. 44 (8), 1067–1083.