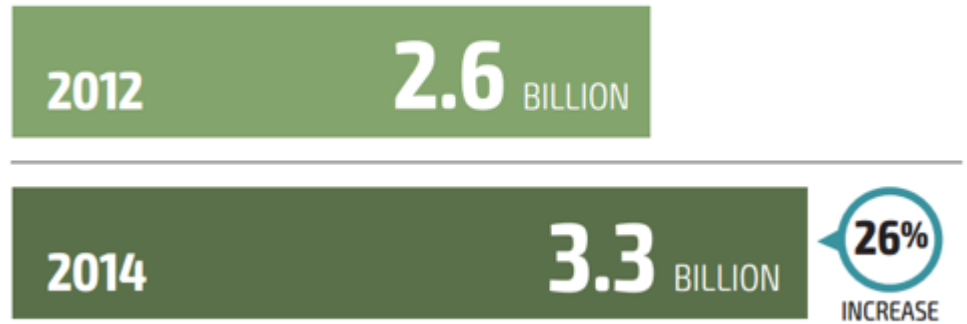
Optimal Discount Policies for Transit Agencies: The Case of Pass-Programs and Loyalty-Programs

Mehdi Nourinejad, Ph.D. Candidate
Amir Gandomi, Professor at Ryerson University
Joseph Y. J. Chow, Professor at New York University
Matthew J. Roorda, Professor at University of Toronto

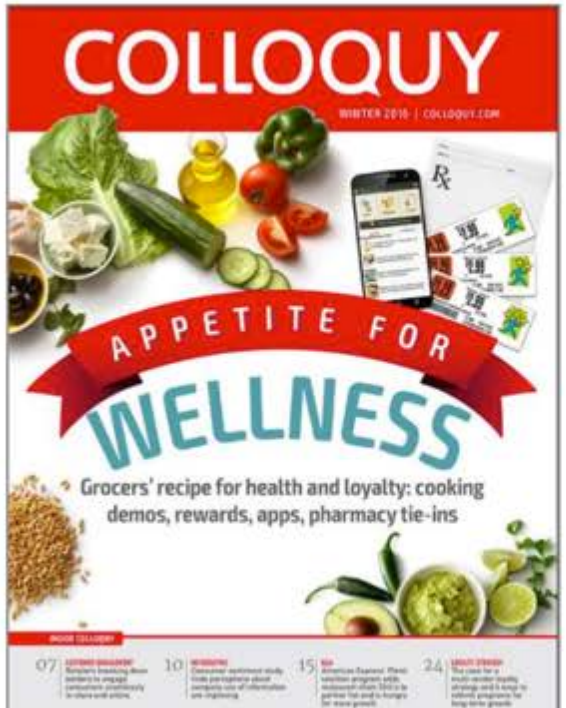
UNIVERSITY OF TORONTO

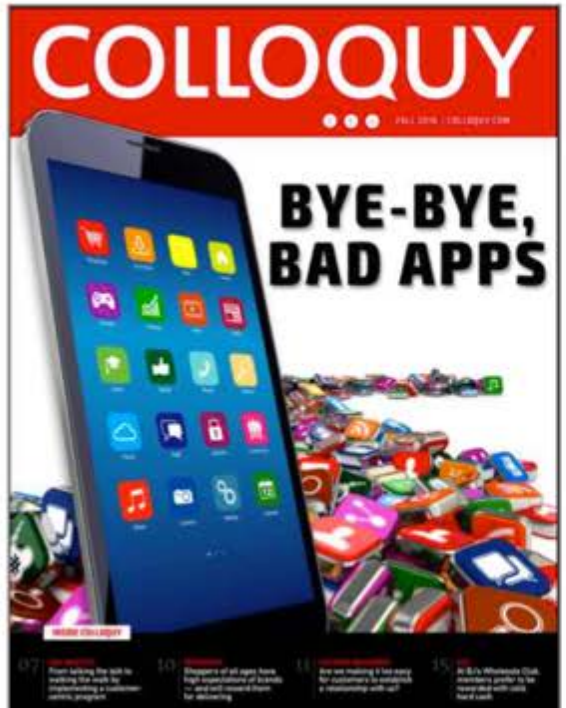
COLLOQUY
Loyalty Talks

Membership Growth: 2012 to 2014

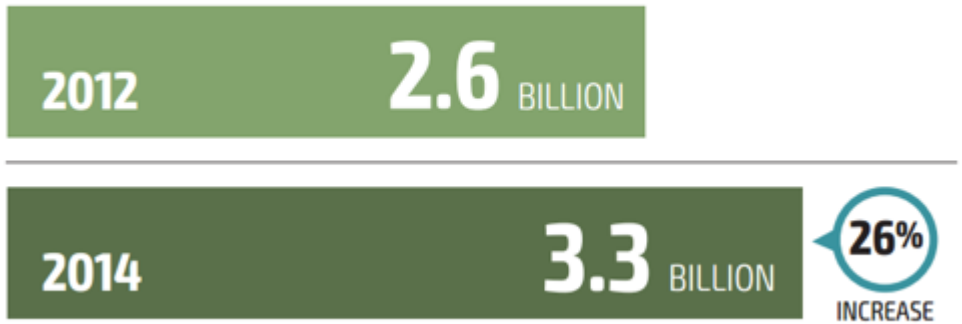


Source: The 2015 COLLOQUY Loyalty Census



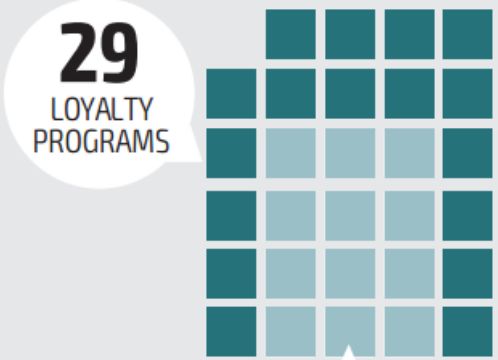


Membership Growth: 2012 to 2014

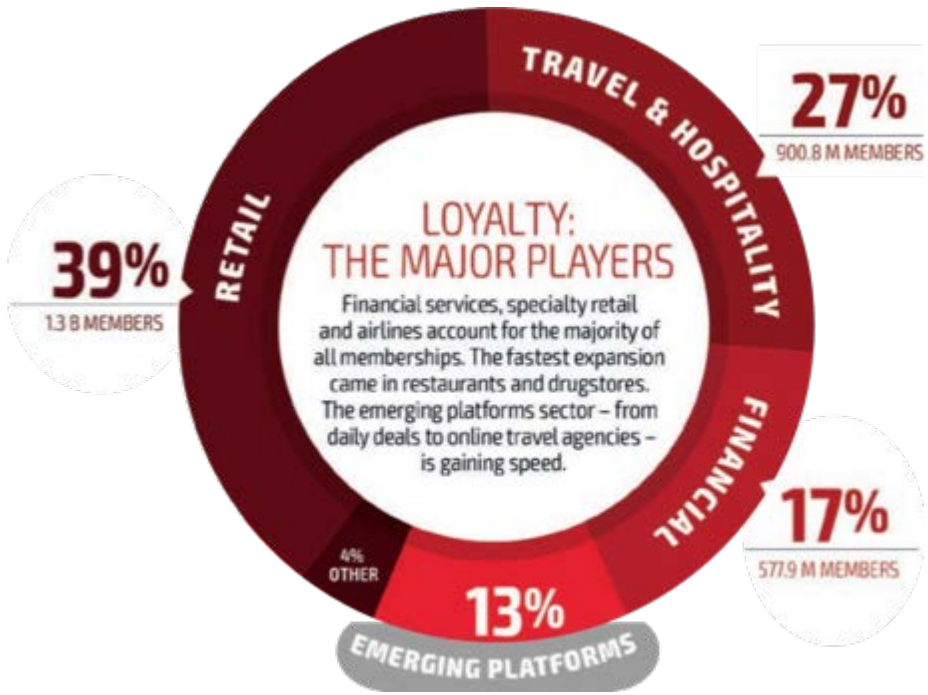


Source: The 2015 COLLOQUY Loyalty Census

Average Household Memberships



Source: The 2015 COLLOQUY Loyalty Census



Loyalty Program in Public Transportation Agencies

NUMBER OF RIDES	DISCOUNT++
1-30	18.40% off**
31-40	95% off**
41+	100% off**

** Fares and discount are estimated and subject to change.*

++ Discount is based on direct routes with no transfers, off a single adult GO fare paper ticket.

*** Actual discount may be .1% lower due to rounding.*

Agency	Adult	Senior	Child	Student	GO co-fare?	Period Pass?	Loyalty Program?
Brampton Transit	$2.90	$1.55	$2.50	$2.50	$0.75	Yes	No
Burlington Transit	$2.70	$1.85	$1.85	$1.85	$0.70	No	Yes[1]
Durham Region Transit	$3.05	$2.00	$2.00	$2.70	$0.75	Yes	No
GO Transit	$5.30	$2.70	$2.70	$5.30	N/A	No	Yes
Hamilton Street Railway	$3.00	$1.80	$1.80	$1.80	$0.50	Yes	Yes
MiWay (Mississauga)	$2.90	$1.90	$1.65	$2.25	$0.80	No	Yes
Oakville Transit	$2.80	$1.80	$2.20	$2.20	$0.75	No	Yes
OC Transpo	$2.84	$2.14	$1.57	$2.84	N/A	Yes	No
Toronto Transit Commission	$2.90	$1.95	free	$1.95	No	Yes	Yes
Union Pearson Express	$9.00	$5.64	free	$9.00	No	Planned	No
York Region Transit	$3.40	$2.10	$2.10	$2.60	$0.75	Yes	Planned

Loyalty Program in Private Transportation Agencies

As a driver on the Lyft platform, you enjoy special access to Accelerate, our driver rewards program. The more rides you give each month, the more you'll reward yourself — and not just when you're behind the wheel.

Silver
0-19 Rides

Gold
20-199 Rides

Platinum
200+ Rides

Research Questions

1. Are loyalty-programs beneficial to transit agencies?
2. Are loyalty-programs better or worse than pass-programs?
3. How to design the discount policy?

Overview

- Literature on loyalty programs
- Motivation
- Pass Programs
- Loyalty Programs
- Comparison between pass and loyalty programs

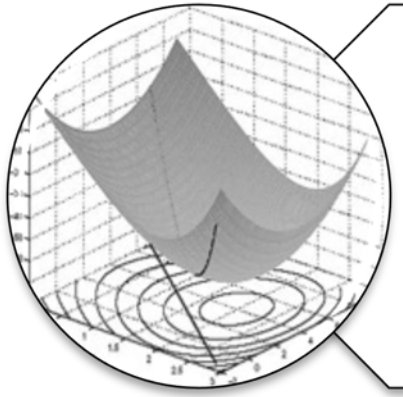
Loyalty Program Literature

Streams of research on loyalty programs

Empirical Studies

take the consumer perspective and explore the effects of LP on customers' buying behavior.

Habib and Hasnine (2017)
McElroy and Miller (2009)



Theoretical Studies

use mathematical modeling to analyze the effects of LP on the firm(s)' profitability and/or market competition.

Our approach

LP Literature

Theoretical Studies

[1] Kim, B. D., Shi, M., & Srinivasan, K. (**2001**). Reward programs and tacit collusion. *Marketing Science*, *20*(2), 99-120.

[2] Lal, R., & Bell, D. E. (**2003**). The impact of frequent shopper programs in grocery retailing. *Quantitative Marketing and Economics*, *1*(2), 179-202.

[3] Kim, B. D., Shi, M., & Srinivasan, K. (2004). Managing capacity through reward programs. *Management Science*, *50*(4), 503-520.

[4] Caminal, R., & Claici, A. (**2007**). Are loyalty-rewarding pricing schemes anti-competitive?. *International Journal of Industrial Organization*, *25*(4), 657-674.

[5] Singh, S. S., Jain, D. C., & Krishnan, T. V. (**2008**). Research Note-Customer Loyalty Programs: Are They Profitable?. *Management Science*, 54(6), 1205-1211.

[6] Caminal, R. (**2012**). The design and efficiency of loyalty rewards. *Journal of Economics & Management Strategy*, *21*(2), 339-371.

[7] Gandomi, A., & Zolfaghari, S. (**2013**). Profitability of loyalty reward programs: An analytical investigation. *Omega*, *41*(4), 797-807.

[8] Sayman, S., & J. Hoch, S. (**2014**). Dynamics of price premiums in loyalty programs. *European Journal of Marketing*, *48*(3/4), 617-640.

[9] Lim, S., & Lee, B. (**2015**). Loyalty programs and dynamic consumer preference in online markets. *Decision Support Systems*, *78*, 104-112.

LP Literature

Study	Market setting	Social welfare included?
[1]	Duopoly	No
[2]	Duopoly	No
[3]	Duopoly	No
[4]	Monopolistic competition/Duopoly	No
[5]	Duopoly	No
[6]	Monopoly	No
[7]	Monopoly	No
[8]	Duopoly	No
[9]	Duopoly	No

Motivation

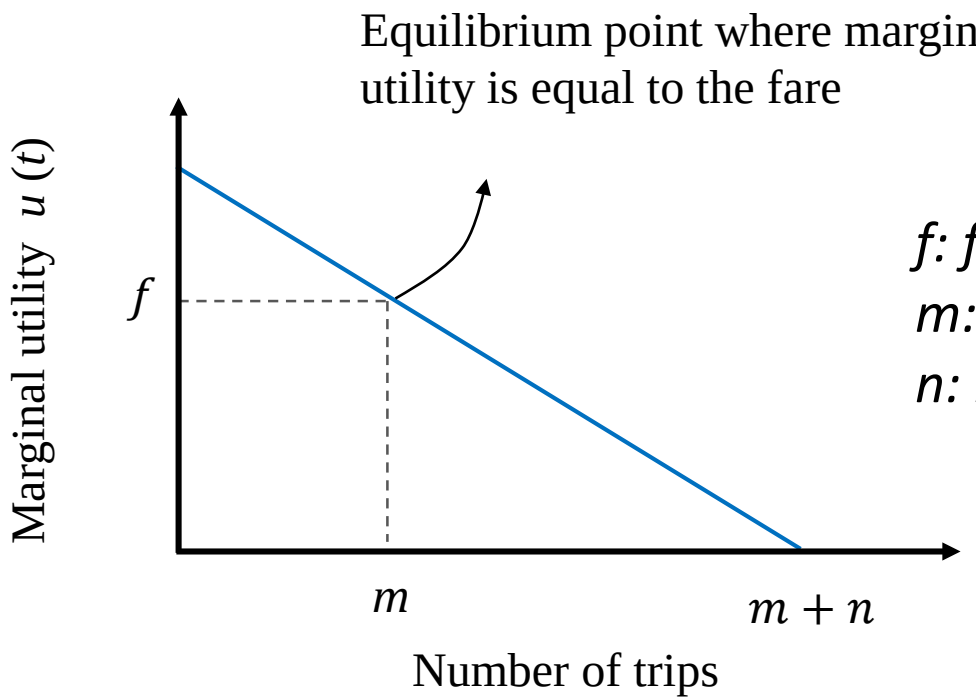
- Growing popularity of loyalty-programs in transit agencies
- Social welfare is not considered in the existing loyalty-program literature
- No comparison between pass-programs and loyalty-programs in terms of profit and social welfare
- Analytical solutions are limited in the loyalty-program literature
- Very few studies on the optimal design of pass-programs
- No studies on the simultaneous presence of pass-programs and loyalty-programs

The Model

Mandatory and Non-mandatory Trips

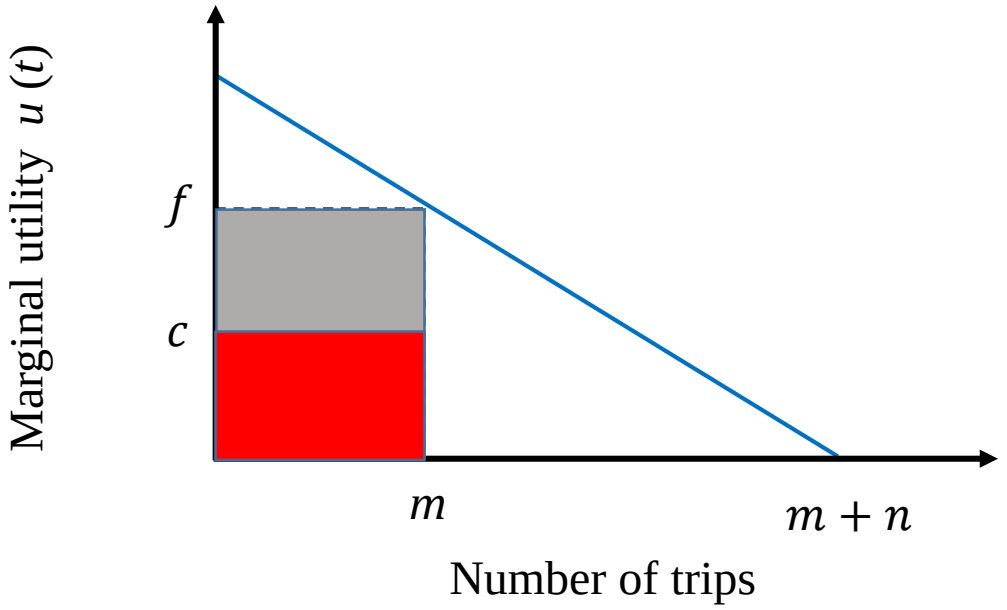
Mandatory Trips

Non-Mandatory Trips



f: fare
m: mandatory trips
n: non-mandatory trips

Profit (without discount policy)



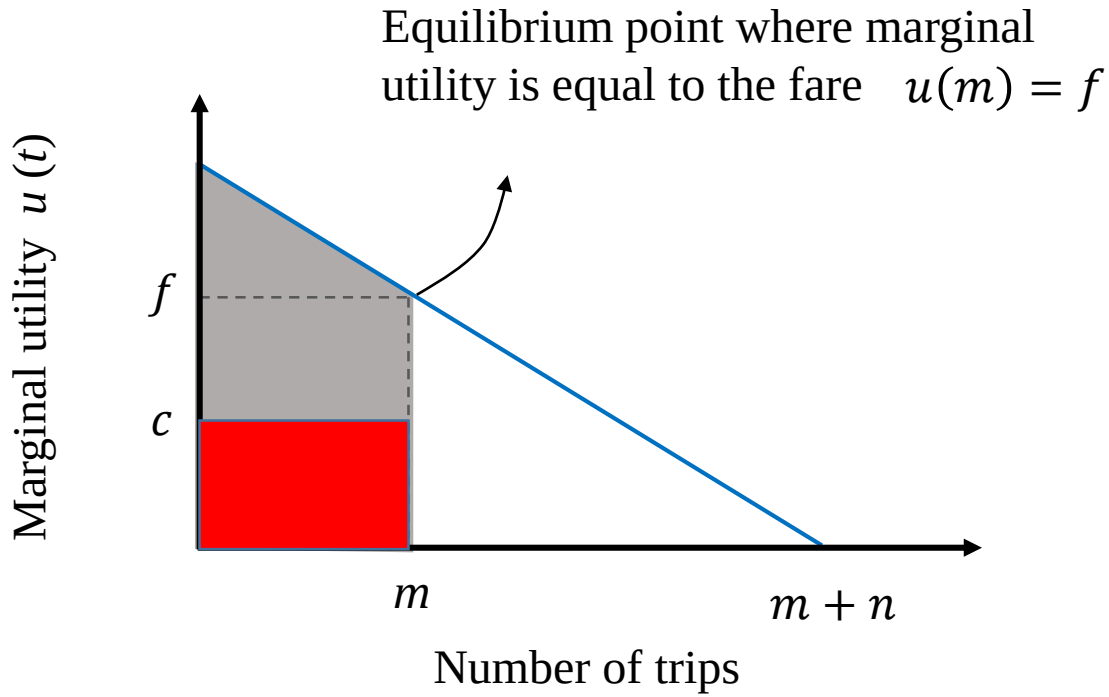
c: Cost of one ride incurred by the transit agency

f: fare

m: mandatory trips

Profit $\pi = fm - cm$

Social Welfare (without discount policy)

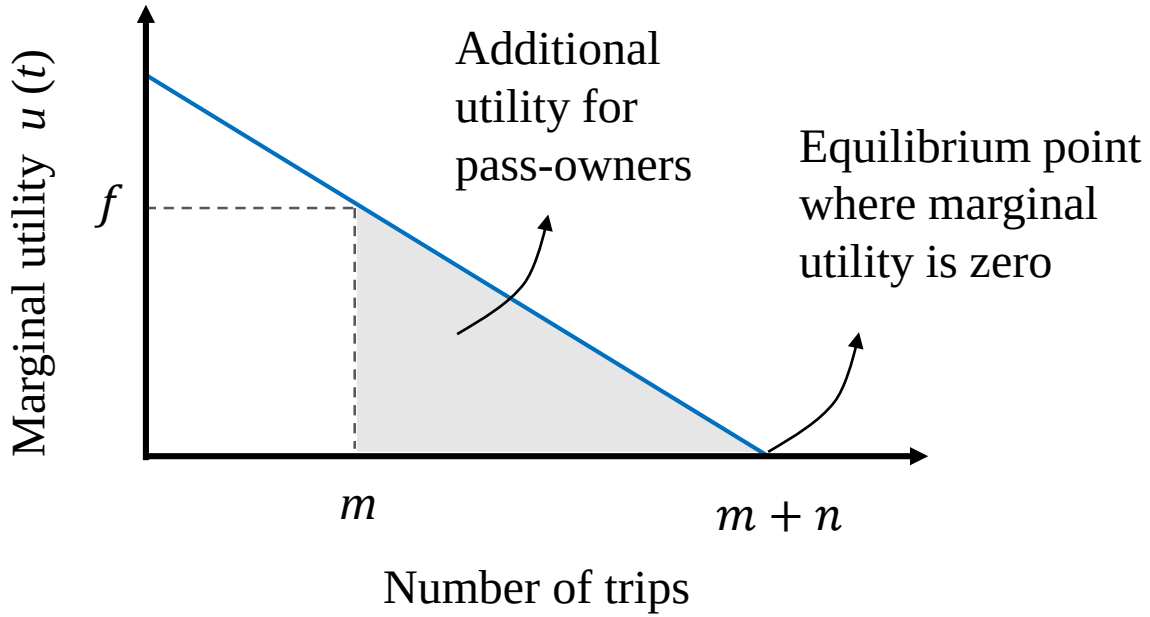


The social welfare : $s = \int_0^m u(t)dt - cm$

The Pass Program

Pass price = $ p

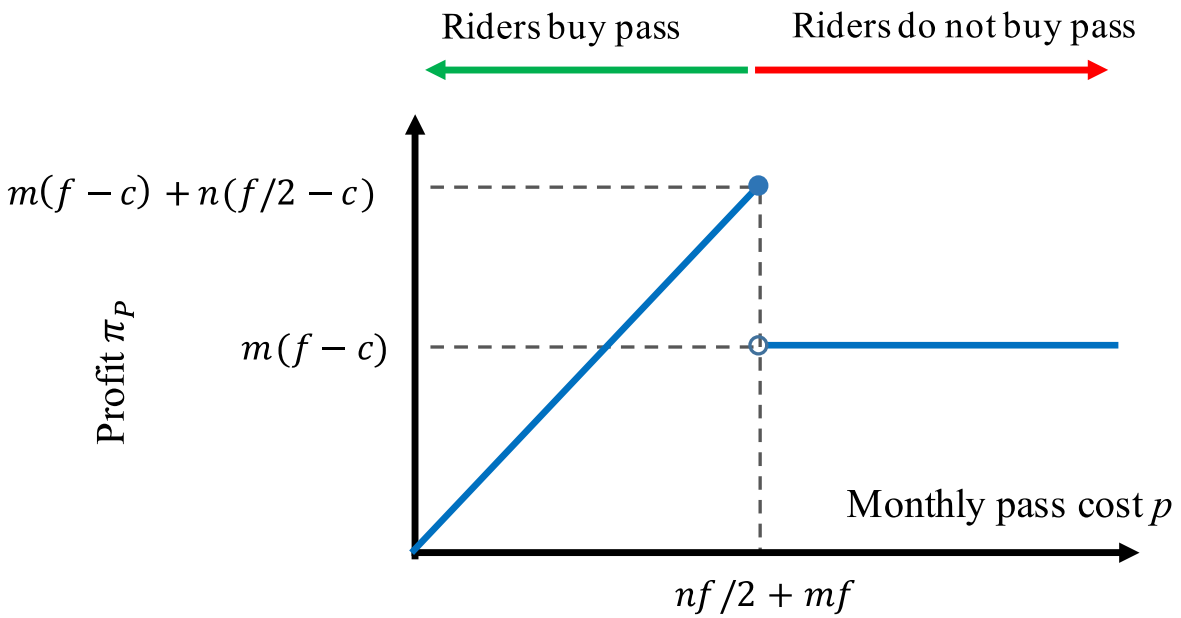
Rider behavior under the pass-policy

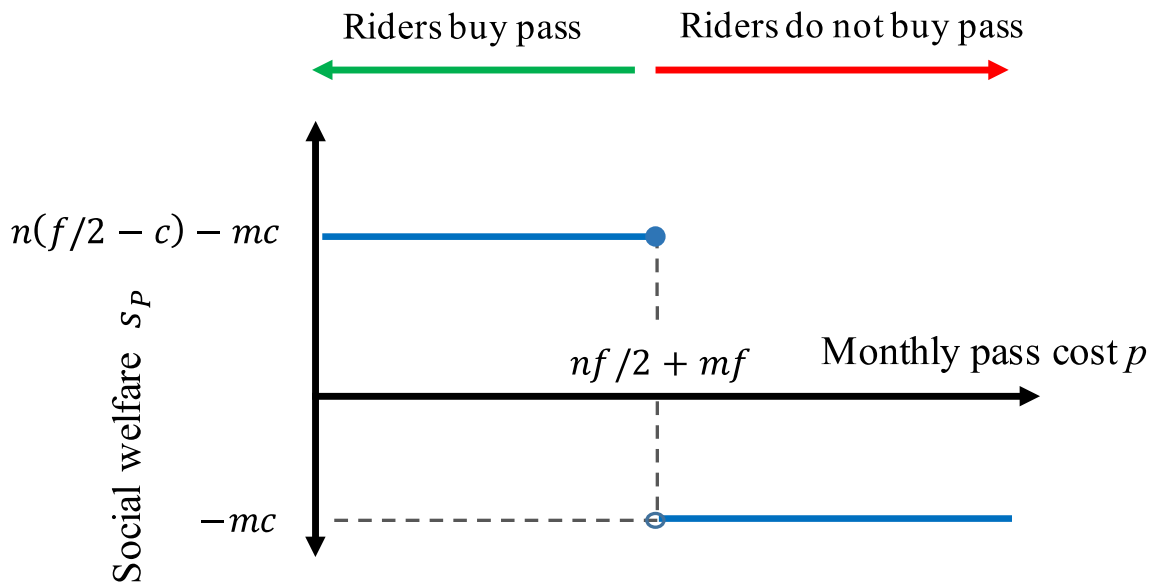


A user only purchases a pass if the cost justifies the benefit $\frac{nf}{2} - p \geq mf$

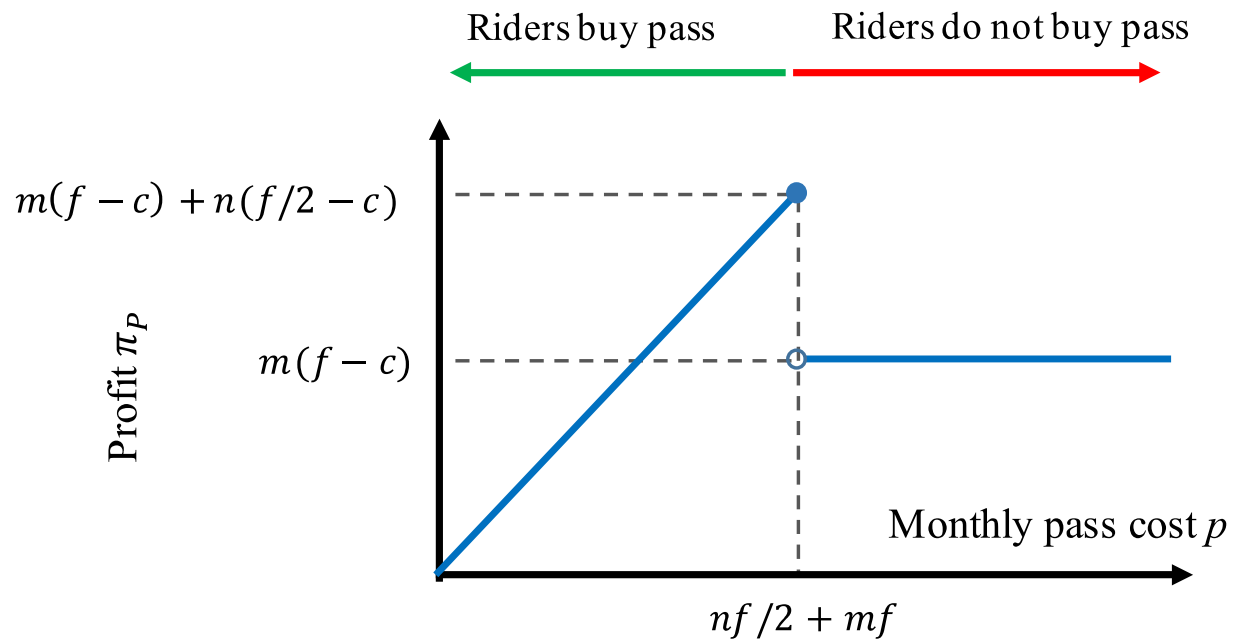
This is equivalent to $\frac{nf}{2} - mf \geq p$

Optimal pass-policy to Maximize Profit/Welfare



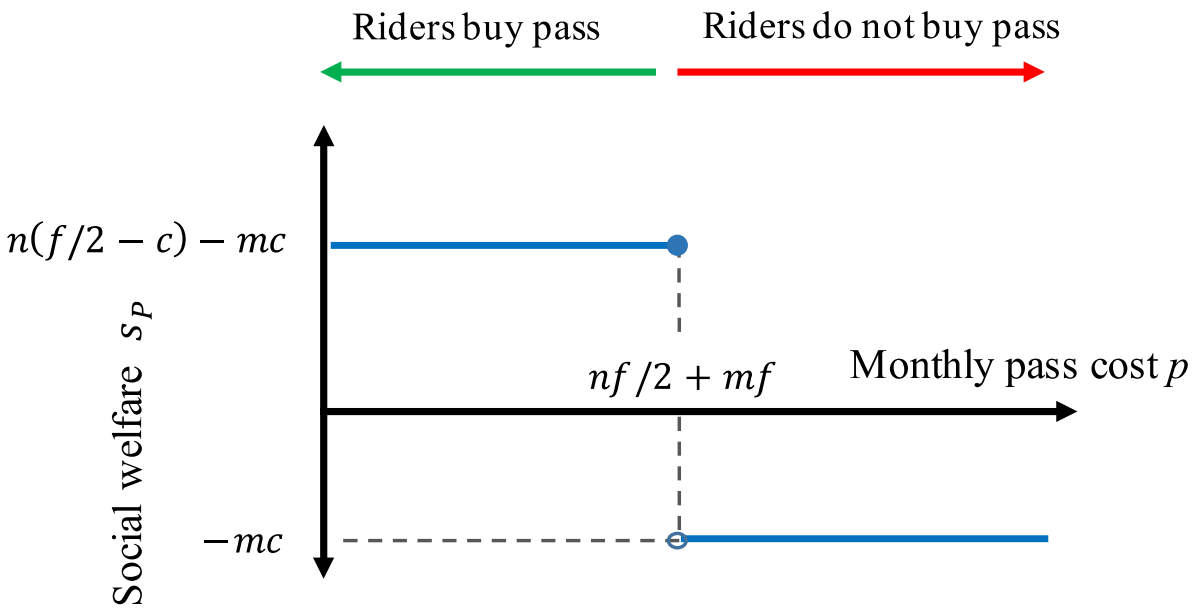


Profit maximization under the pass-policy



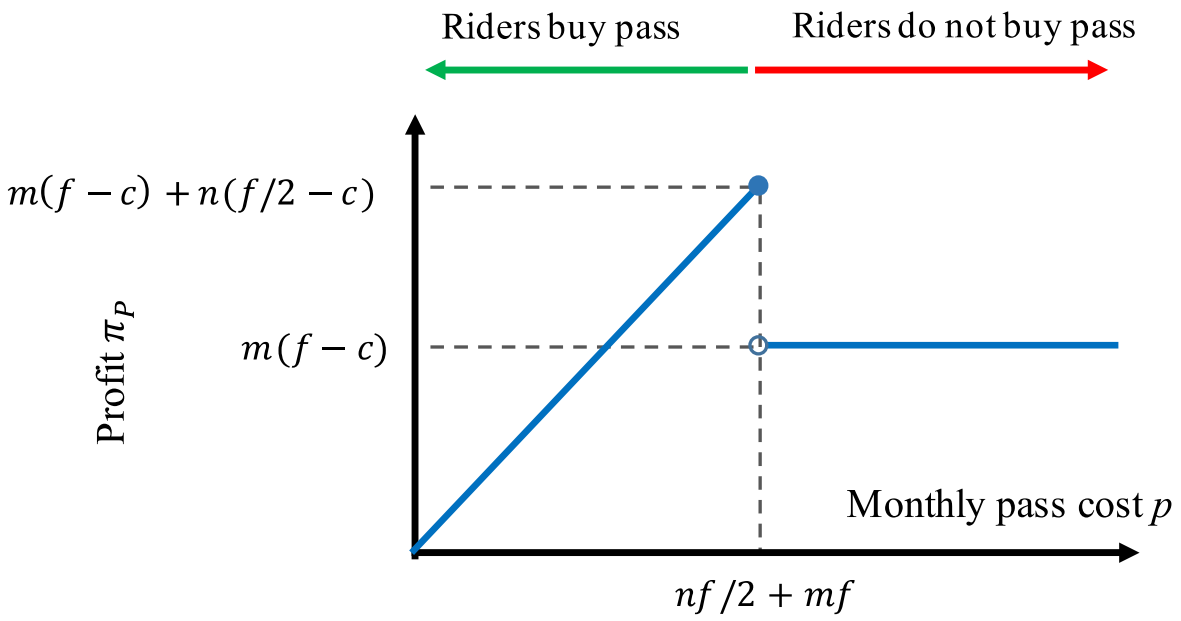
The pass-program improves profit if $c < \frac{f}{2}$

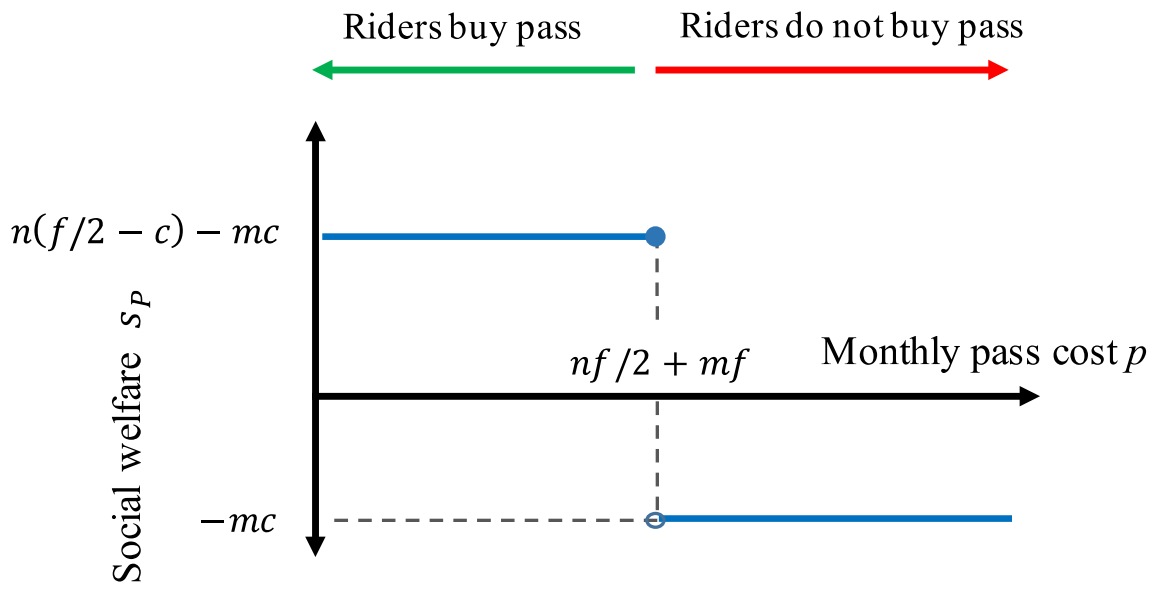
Social welfare maximization under the pass-policy



The pass-program improves social welfare if is only viable when $c < f/2$.

Optimal pass-policy to Maximize Profit/Welfare





First-best and second-best solutions are obtained at the same pass price.

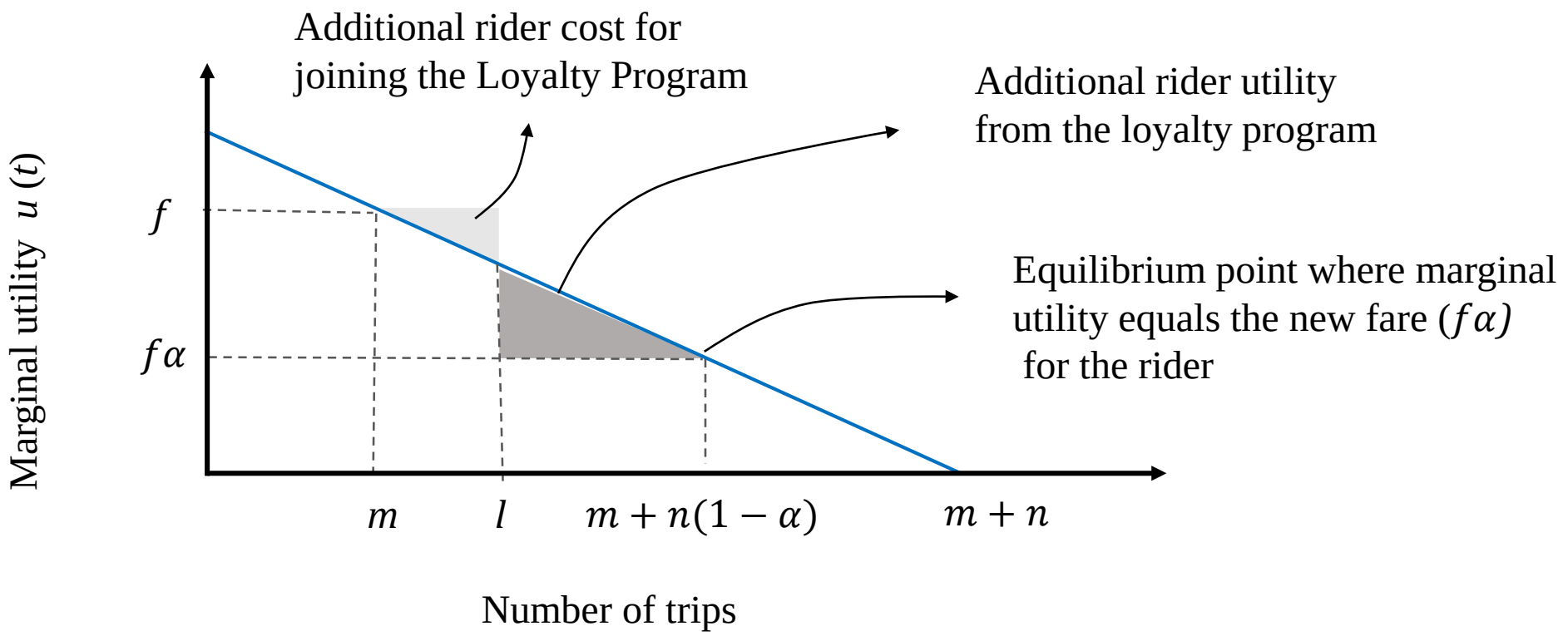
Loyalty-Program

Users get a discount of α (i.e., they pay αf dollars per trip) after completing a total of l trips.

NUMBER OF RIDES	DISCOUNT++
1-30	18.40% off**
31-40	95% off**
41+	100% off**

** Fares and discount are estimated and subject to change.*
++ Discount is based on direct routes with no transfers, off a single adult GO fare paper ticket.
*** Actual discount may be .1% lower due to rounding.*

User behavior under loyalty-program



A rider will only use the loyalty program if $l \leq m + (1-\alpha)n/2$

Profit maximization under the loyalty program

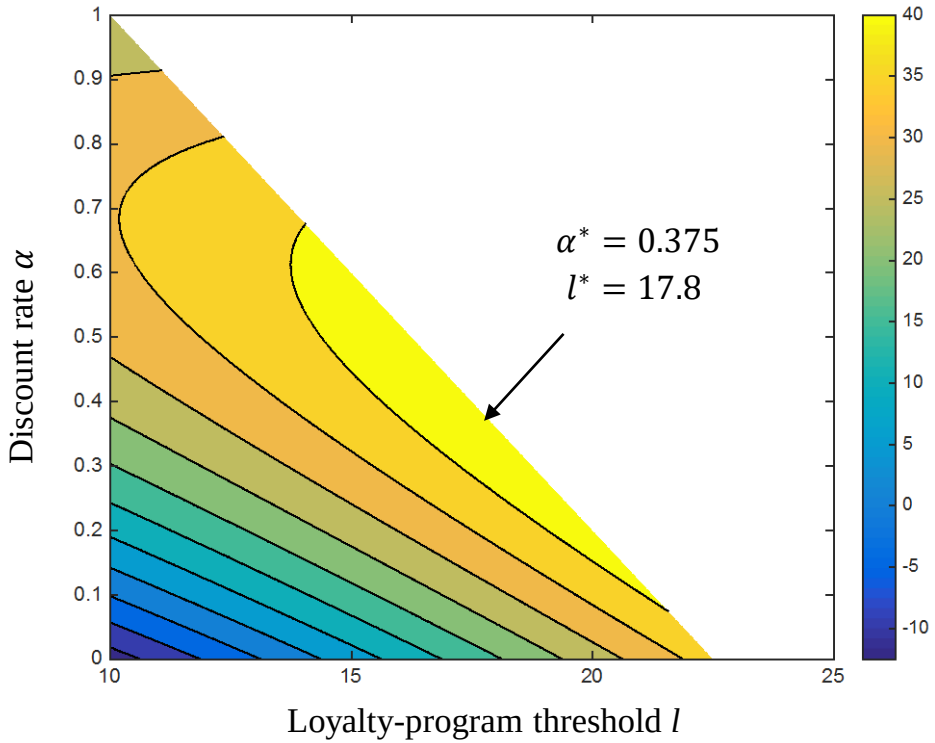
$$\pi_L = lf + \alpha f[m + n(1-\alpha) - l] - c_L[m + n(1-\alpha)]$$

The function π_L is strictly concave, so it is maximized at a unique solution (α^*, l^*).

The optimal discount rate for profit maximization is $\alpha^* = c_L/f$.

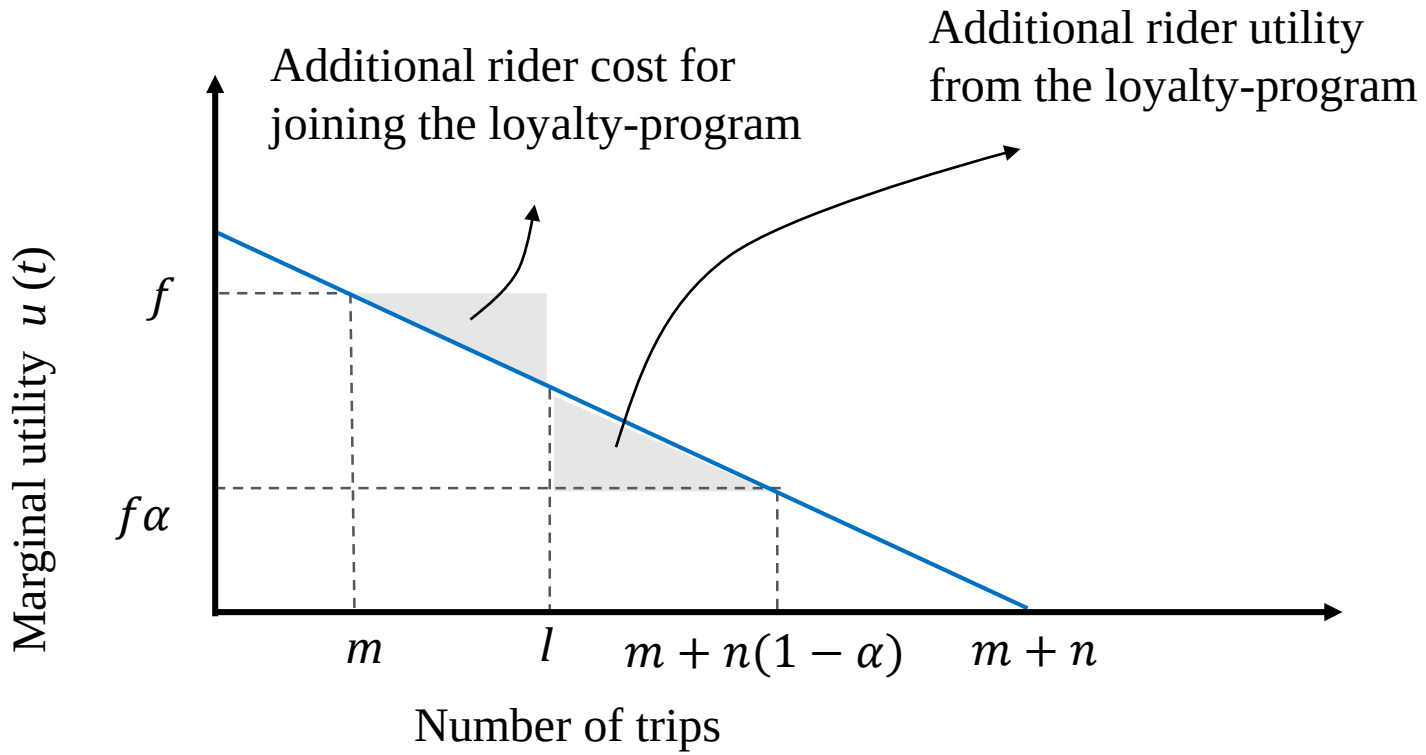
The optimal discount rate for profit maximization is $l^* = m + (1 - c_L/p)n/2$.

The optimal profit of the loyalty program is $\pi_L^* = \left[m + \frac{\left(1-\frac{c_L}{f}\right)n}{2}\right](f - c_L)$



$m = 10; n = 25; f = 4; c = 1.5$

Social-welfare maximization under the loyalty program



$$s_L = f(1-\alpha)[m + n(1-\alpha)/2 - l] - [m + n(1-\alpha)]c_L$$

Social-welfare maximization under the loyalty program

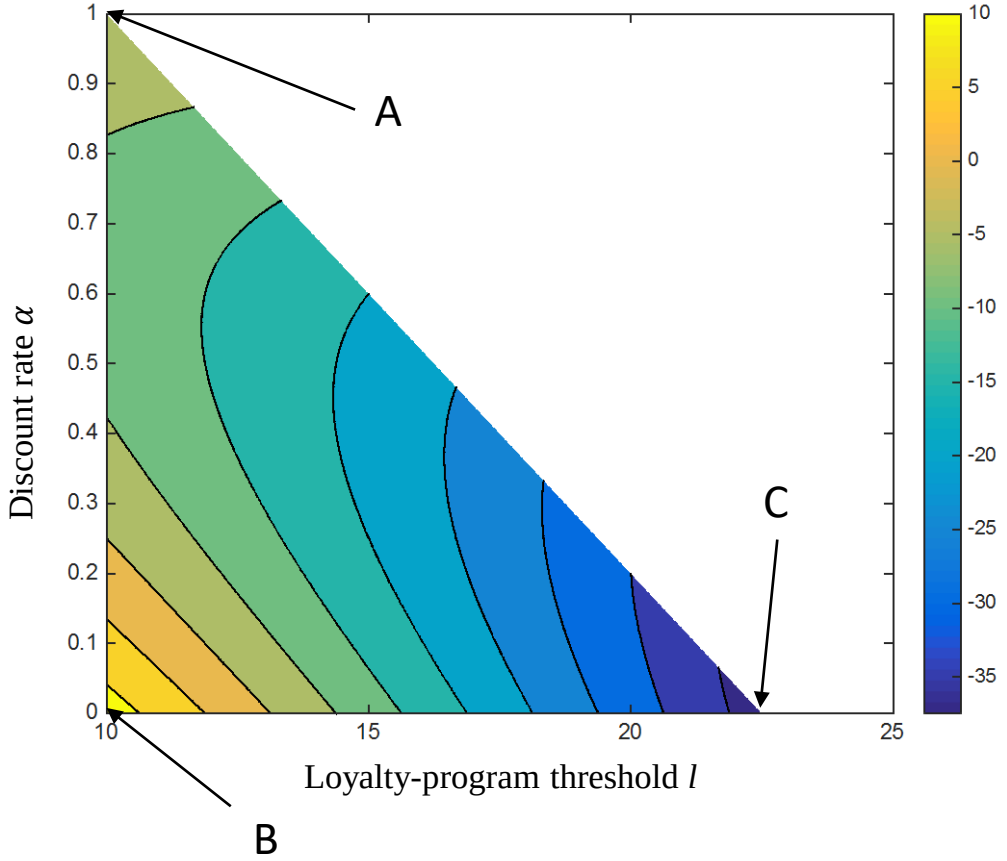
$$s_L = f(1-\alpha)[m + n(1-\alpha)/2 - l] - [m + n(1-\alpha)]c_L$$

Function $s_L(\alpha, l)$ is strictly convex.
Given that we want to maximize s_L, the optimal solution (α°, l°) falls on the boundaries.

Point A: $(\alpha, l) = (1, m) \longrightarrow s_L(\alpha, l) = -mc_L$

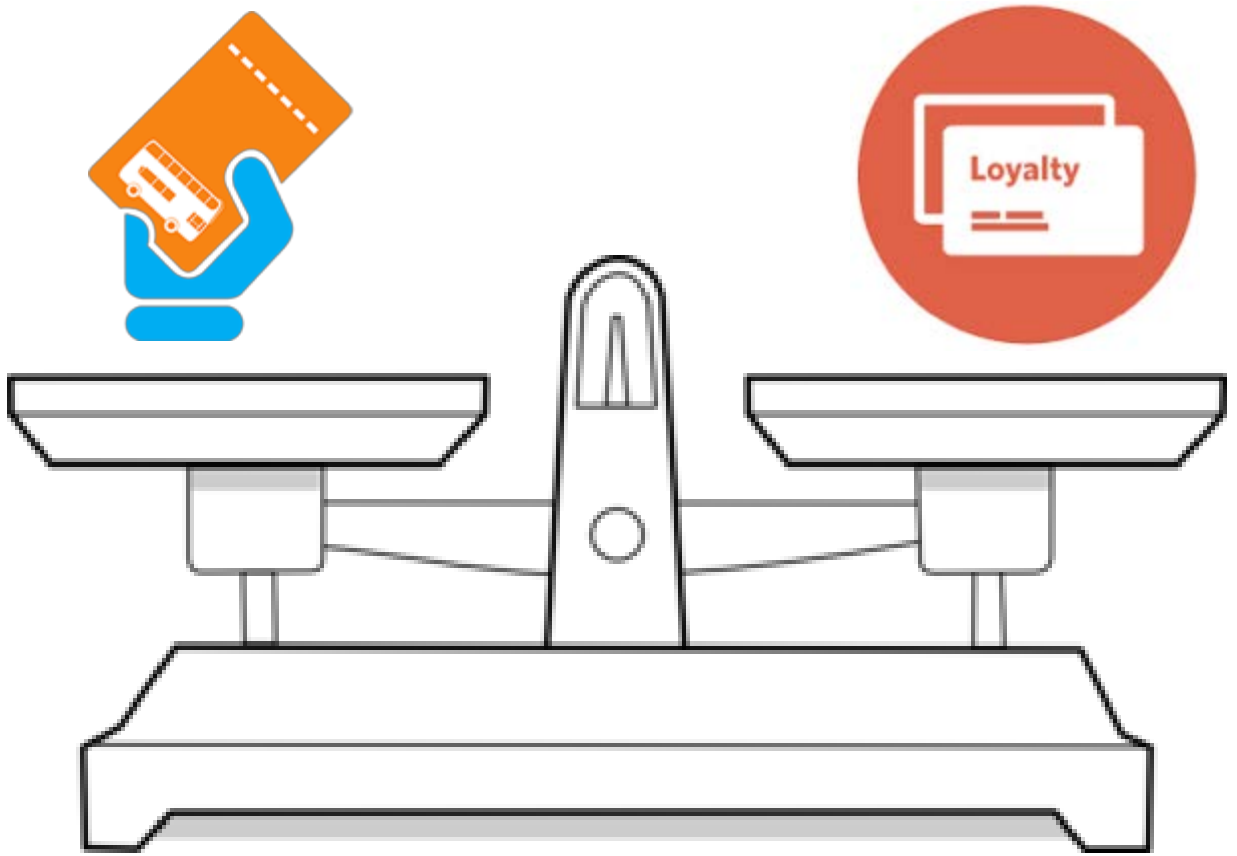
Point B: $(\alpha, l) = (0, m) \longrightarrow s_L(\alpha, l) = \frac{nf}{2} - (m+n)c_L$

It is clear that point B has a higher social welfare. Hence, $(\alpha^\circ, l^\circ) = (0, m)$ and $s_L^\circ = \frac{nf}{2} - (m+n)c_L$



$m = 10; n = 25; f = 4; c = 1.5$

Comparison Between the Loyalty Program and the Pass Program

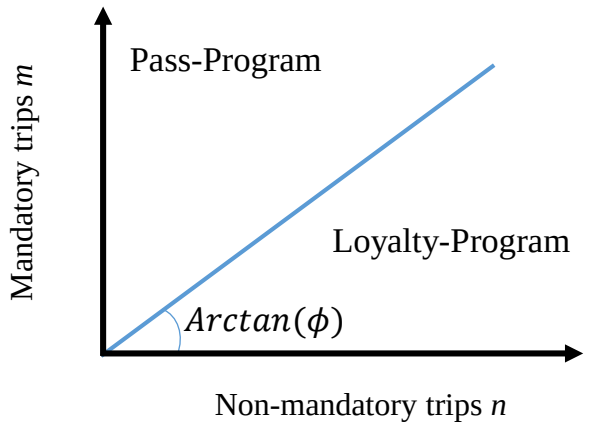


Comparison of Profit

$$\pi_L^* = m(f - c_L) + n\left[\frac{\left(1 - \frac{c_L}{f}\right)}{2}\right](f - c_L) \quad \leq \quad \pi_P^* = m(f - c) + n(f/2 - c)$$

The loyalty program generates higher profit than the pass-program if and only if $m/n \leq \phi(c_L, c_L, f)$ where

$$\phi(c_L, c_L, f) = \frac{(f - c_L)^2 - f^2 + 2fc}{2f(c_L - c)} \equiv \frac{{c_L}^2}{2f(c_L - c)} - 1.$$



Comparative analysis of the social-welfare in the Loyalty-Program and the Pass-Policy

$$s_L^{\circ} = \frac{nf}{2} - (m+n)\boxed{c_L}$$

$$s_P^{\circ} = \frac{nf}{2} - (m+n)\boxed{c}$$

The optimal social-welfare from the pass-program is always higher than the loyalty-program.

Analysis of Existing Pass Programs and Loyalty Programs

Burlington

Policy 1: $l_{monthly} = \frac{p_{monthly}}{f}$, $l_{weekly} = \frac{p_{weekly}}{f}$

Age	Tickets	Monthly Pass
Adults	10 / $27.50	$97.00
Students	10 / $19.00	$71.00
Seniors	10 / $19.00	$59.25
Children	10 / $18.50	

Age	PRESTO Price
Adults	$2.70
Students	$1.85
Seniors	$1.85
Children	$1.85

Adults	Travel free after **36** single fare rides in same calendar month
Students	Travel free after **38** single fare rides in same calendar month
Seniors	Travel free after **32** single fare rides in same calendar month
Children	Travel free after **38** single-fare rides in the same calendar month

97.00/2.70=	35.93
71.00/1.85=	38.38
59.25/1.85=	32.03

Hamilton

Policy 2: $l_{weekly} = \frac{p_{monthly}/4}{f}$, $l_{weekly} = \frac{p_{monthly}/4.33}{f}$

Example:

Fare class	Single PRESTO fare	Weekly frequent rider discount	PRESTO Passes
Adult	$2.30	Free after 11 PRESTO trips in same week (Monday to Sunday)	Monthly: $101.20
Child	$1.90	Free after 11 PRESTO trips in same week (Monday to Sunday)	Monthly: $83.60
Student	$1.90	Free after 11 PRESTO trips in same week (Monday to Sunday)	Monthly: $83.60
Senior	$1.90	Free after **11** PRESTO trips in same week (Monday to Sunday)	Monthly:$26.50

(101.20/4)/2.30 =	11.00
(83.60/4)/1.90 =	11.00
(83.60/4)/1.90 =	11.00
(26.50/4)/1.90 =	**3.49**

Mississauga- MiWay

Policy 3: Set l and m independently.

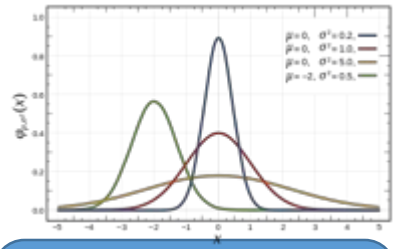
Example:

Fare class	Single PRESTO fare	Weekly frequent rider discount	PRESTO Passes
Adult	$3.00	Free after **12** full-fare trips in same week (Mon. to Sun.)	Monthly: $130
Child	$1.65	Free after **12** full-fare trips in same week (Mon. to Sun.)	-
High School Student	$2.25	Free after **12** full-fare trips in same week (Mon. to Sun.)	-
Post-Secondary Student	$2.85	Free after **12** full-fare trips in same week (Mon. to Sun.)	-
Senior	$2.00	Free after **12** full-fare trips in same week (Mon. to Sun.)	Monthly: $61

(130.00/4.00)/3.00 =	10.83
(130.00/4.33)/3.00 =	10.01
(61.00/4.00)/2.00 =	7.63
(61.00/4.33)/2.00=	7.03

$\neq 12$

Simulation Model for Complex Cases



Rider Generation

Compute the total benefit of each option

Rider picks an option

Compute social welfare and profit

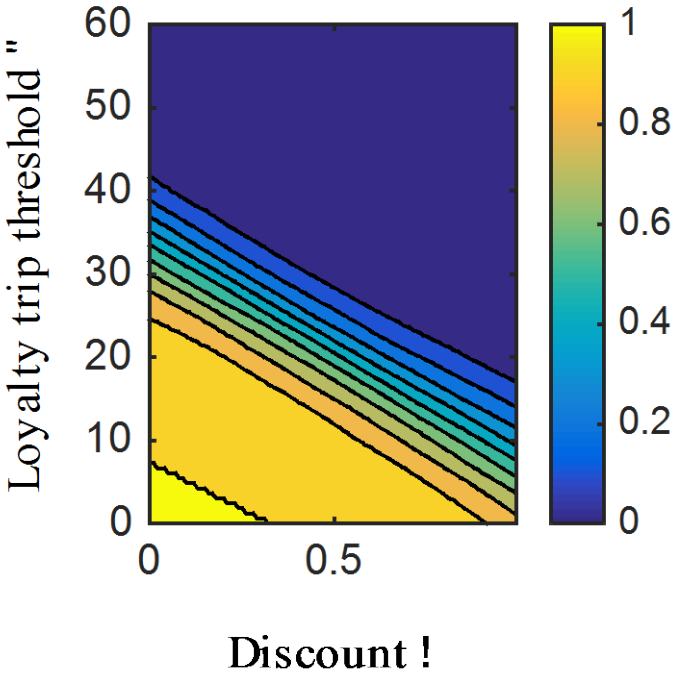
$$G_P = nf/2 - p$$
$$G_L = f(1-\alpha)(m + n(1-\alpha)/2 - l) + lf - \alpha f[m + n(1-\alpha) - l]$$
$$G = -mf$$

Simulation Results: Pass Program

yer ratio

Simulation Results: Loyalty Program

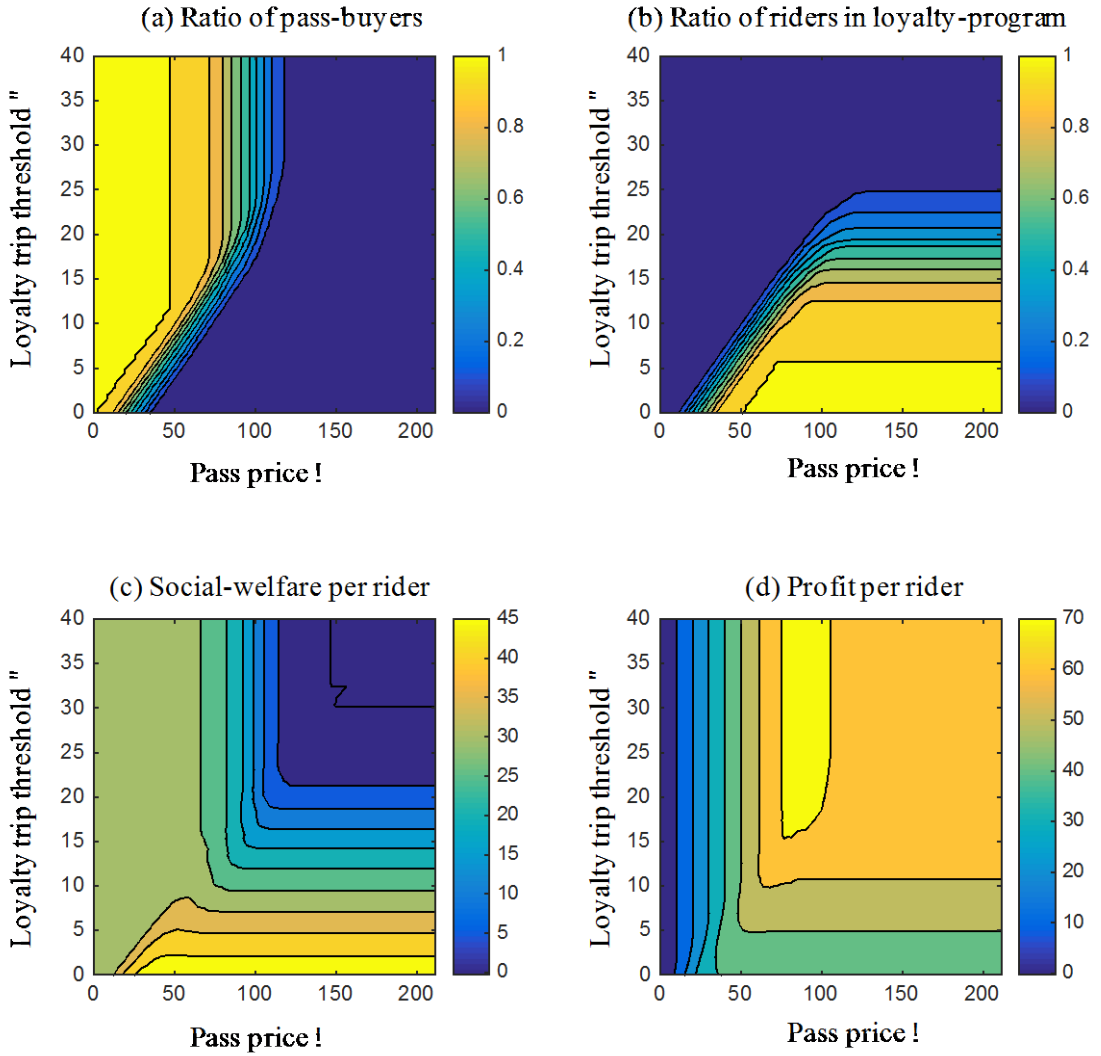
Ratio of riders in loyalty-program



Profit per rider

Social-welfare per rider

Both Programs are Offered



Key findings

- Pass-policy is viable only when the cost per user is lower than half the fare
- Pass-policy simultaneously maximizes social welfare and profit
- First-best and second-best social welfare solutions coincide in the pass-program
- The optimal discount rate in the loyalty-program is ratio of cost (per user) over fare for profit maximization and it is equal to zero for welfare maximization
- The optimal discount rate in the loyalty-program is zero for welfare maximization
- Profit is generated in the loyalty program only from the first l trips (i.e., trip threshold after which the users get a discount)
- According to the ratio m/n (mandatory over non-mandatory trips) one of the discount-policies generates higher profit
- The pass-program always generates higher social-welfare than the loyalty program

Future research

- Multi-tier loyalty programs
- Crowding costs
- Peak and off-peak periods\spatial structure of the transit network
- Risk-behavior
- Empirical validation